CHAPTER 7
EXAMPLES: MIXTURE MODELING WITH CROSS-SECTIONAL DATA

Mixture modeling refers to modeling with categorical latent variables that represent subpopulations where population membership is not known but is inferred from the data. This is referred to as finite mixture modeling in statistics (McLachlan & Peel, 2000). A special case is latent class analysis (LCA) where the latent classes explain the relationships among the observed dependent variables similar to factor analysis. In contrast to factor analysis, however, LCA provides classification of individuals. In addition to conventional exploratory LCA, confirmatory LCA and LCA with multiple categorical latent variables can be estimated. In Mplus, mixture modeling can be applied to any of the analyses discussed in the other example chapters including regression analysis, path analysis, confirmatory factor analysis (CFA), item response theory (IRT) analysis, structural equation modeling (SEM), growth modeling, survival analysis, and multilevel modeling. Observed dependent variables can be continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types. LCA and general mixture models can be extended to include continuous latent variables. An overview can be found in Muthén (2008).

LCA is a measurement model. A general mixture model has two parts: a measurement model and a structural model. The measurement model for LCA and the general mixture model is a multivariate regression model that describes the relationships between a set of observed dependent variables and a set of categorical latent variables. The observed dependent variables are referred to as latent class indicators. The relationships are described by a set of linear regression equations for continuous latent class indicators, a set of censored normal or censored-inflated normal regression equations for censored latent class indicators, a set of logistic regression equations for binary or ordered categorical latent class indicators, a set of multinomial logistic regressions for unordered categorical latent class indicators, and a set of Poisson or

zero-inflated Poisson regression equations for count latent class indicators.

The structural model describes three types of relationships in one set of multivariate regression equations: the relationships among the categorical latent variables, the relationships among observed variables, and the relationships between the categorical latent variables and observed variables that are not latent class indicators. These relationships are described by a set of multinomial logistic regression equations for the categorical latent dependent variables and unordered observed dependent variables, a set of linear regression equations for continuous observed dependent variables, a set of censored normal or censored normal regression equations for censored-inflated observed dependent variables, a set of logistic regression equations for binary or ordered categorical observed dependent variables, and a set of Poisson or zero-inflated Poisson regression equations for count observed dependent variables. For logistic regression, ordered categorical variables are modeled using the proportional odds specification. Maximum likelihood estimation is used.

The general mixture model can be extended to include continuous latent variables. The measurement and structural models for continuous latent variables are described in Chapter 5. In the extended general mixture model, relationships between categorical and continuous latent variables are allowed. These relationships are described by a set of multinomial logistic regression equations for the categorical latent dependent variables and a set of linear regression equations for the continuous latent dependent variables.

In mixture modeling, some starting values may result in local solutions that do not represent the global maximum of the likelihood. To avoid this, different sets of starting values are automatically produced and the solution with the best likelihood is reported.

All cross-sectional mixture models can be estimated using the following special features:

- Single or multiple group analysis
- Missing data
- Complex survey data

- Latent variable interactions and non-linear factor analysis using maximum likelihood
- Random slopes
- Linear and non-linear parameter constraints
- Indirect effects including specific paths
- Maximum likelihood estimation for all outcome types
- Bootstrap standard errors and confidence intervals
- Wald chi-square test of parameter equalities
- Test of equality of means across latent classes using posterior probability-based multiple imputations

For TYPE=MIXTURE, multiple group analysis is specified by using the KNOWNCLASS option of the VARIABLE command. The default is to estimate the model under missing data theory using all available data. The LISTWISE option of the DATA command can be used to delete all observations from the analysis that have missing values on one or more of the analysis variables. Corrections to the standard errors and chi-square test of model fit that take into account stratification, non-independence of observations, and unequal probability of selection are obtained by using the TYPE=COMPLEX option of the ANALYSIS command in conjunction with the STRATIFICATION, CLUSTER, and WEIGHT options of the VARIABLE command. The SUBPOPULATION option is used to select observations for an analysis when a subpopulation (domain) is analyzed. Latent variable interactions are specified by using the | symbol of the MODEL command in conjunction with the XWITH option of the MODEL command. Random slopes are specified by using the | symbol of the MODEL command in conjunction with the ON option of the MODEL command. Linear and non-linear parameter constraints are specified by using the MODEL CONSTRAINT command. Indirect effects are specified by using the MODEL INDIRECT command. Maximum likelihood estimation is specified by using the ESTIMATOR option of the ANALYSIS command. Bootstrap standard errors are obtained by using the BOOTSTRAP option of the ANALYSIS command. Bootstrap confidence intervals are obtained by using the BOOTSTRAP option of the ANALYSIS command in conjunction with the CINTERVAL option of the OUTPUT command. The MODEL TEST command is used to test linear restrictions on the parameters in the MODEL and MODEL CONSTRAINT commands using the Wald chi-square test. The AUXILIARY option is used to test the equality of means across latent classes using posterior probability-based multiple imputations.

Graphical displays of observed data and analysis results can be obtained using the PLOT command in conjunction with a post-processing graphics module. The PLOT command provides histograms, scatterplots, plots of individual observed and estimated values, plots of sample and estimated means and proportions/probabilities, and plots of estimated probabilities for a categorical latent variable as a function of its covariates. These are available for the total sample, by group, by class, and adjusted for covariates. The PLOT command includes a display showing a set of descriptive statistics for each variable. The graphical displays can be edited and exported as a DIB, EMF, or JPEG file. In addition, the data for each graphical display can be saved in an external file for use by another graphics program.

Following is the set of examples included in this chapter.

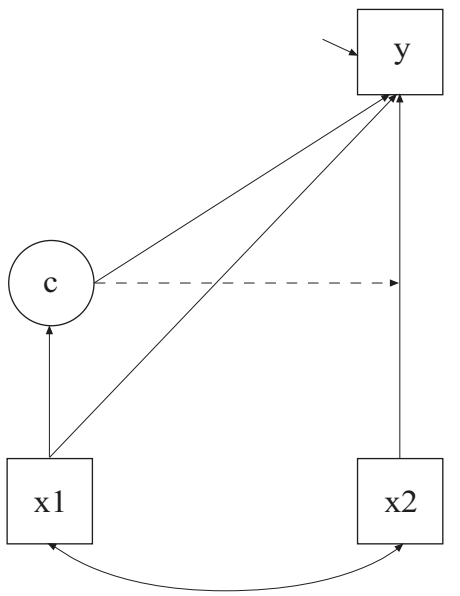
- 7.1: Mixture regression analysis for a continuous dependent variable using automatic starting values with random starts
- 7.2: Mixture regression analysis for a count variable using a zero-inflated Poisson model using automatic starting values with random starts
- 7.3: LCA with binary latent class indicators using automatic starting values with random starts
- 7.4: LCA with binary latent class indicators using user-specified starting values without random starts
- 7.5: LCA with binary latent class indicators using user-specified starting values with random starts
- 7.6: LCA with three-category latent class indicators using user-specified starting values without random starts
- 7.7: LCA with unordered categorical latent class indicators using automatic starting values with random starts
- 7.8: LCA with unordered categorical latent class indicators using user-specified starting values with random starts
- 7.9: LCA with continuous latent class indicators using automatic starting values with random starts
- 7.10: LCA with continuous latent class indicators using user-specified starting values without random starts
- 7.11: LCA with binary, censored, unordered, and count latent class indicators using user-specified starting values without random starts
- 7.12: LCA with binary latent class indicators using automatic starting values with random starts with a covariate and a direct effect

- 7.13: Confirmatory LCA with binary latent class indicators and parameter constraints
- 7.14: Confirmatory LCA with two categorical latent variables
- 7.15: Loglinear model for a three-way table with conditional independence between the first two variables
- 7.16: LCA with partial conditional independence*
- 7.17: Mixture CFA modeling
- 7.18: LCA with a second-order factor (twin analysis)*
- 7.19: SEM with a categorical latent variable regressed on a continuous latent variable*
- 7.20: Structural equation mixture modeling
- 7.21: Mixture modeling with known classes (multiple group analysis)
- 7.22: Mixture modeling with continuous variables that correlate within class
- 7.23: Mixture randomized trials modeling using CACE estimation with training data
- 7.24: Mixture randomized trials modeling using CACE estimation with missing data on the latent class indicator
- 7.25: Zero-inflated Poisson regression carried out as a two-class model
- 7.26: CFA with a non-parametric representation of a non-normal factor distribution
- 7.27: Factor (IRT) mixture analysis with binary latent class and factor indicators*
- 7.28: Two-group twin model for categorical outcomes using maximum likelihood and parameter constraints*
- 7.29: Two-group IRT twin model for factors with categorical factor indicators using parameter constraints*
- 7.30: Continuous-time survival analysis using a Cox regression model to estimate a treatment effect

* Example uses numerical integration in the estimation of the model. This can be computationally demanding depending on the size of the problem.

EXAMPLE 7.1: MIXTURE REGRESSION ANALYSIS FOR A CONTINUOUS DEPENDENT VARIABLE USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a mixture regression
           analysis for a continuous dependent
           variable using automatic starting values
           with random starts
DATA:      FILE IS ex7.1.dat;
VARIABLE:  NAMES ARE y x1 x2;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           y ON x1 x2;
           c ON x1;
           %c#2%
           y ON x2;
           y;
OUTPUT:    TECH1 TECH8;
```



In this example, the mixture regression model for a continuous dependent variable shown in the picture above is estimated using automatic starting values with random starts. Because c is a categorical latent variable, the interpretation of the picture is not the same as for models with continuous latent variables. The arrow from c to y indicates that the intercept of y varies across the classes of c. This corresponds to the regression of y on a set of dummy variables representing the categories of c. The broken arrow from c to the arrow from x2 to y indicates that the slope in the regression of y on x2 varies across the classes of c. The arrow from x1 to c represents the multinomial logistic regression of c on x1.

```
TITLE:      this is an example of a mixture regression
            analysis for a continuous dependent
            variable
```

The TITLE command is used to provide a title for the analysis. The title is printed in the output just before the Summary of Analysis.

```
DATA:       FILE IS ex7.1.dat;
```

The DATA command is used to provide information about the data set to be analyzed. The FILE option is used to specify the name of the file that contains the data to be analyzed, ex7.1.dat. Because the data set is in free format, the default, a FORMAT statement is not required.

```
VARIABLE:   NAMES ARE y x1 x2;
            CLASSES = c (2);
```

The VARIABLE command is used to provide information about the variables in the data set to be analyzed. The NAMES option is used to assign names to the variables in the data set. The data set in this example contains three variables: y, x1, and x2. The CLASSES option is used to assign names to the categorical latent variables in the model and to specify the number of latent classes in the model for each categorical latent variable. In the example above, there is one categorical latent variable c that has two latent classes.

```
ANALYSIS:   TYPE = MIXTURE;
```

The ANALYSIS command is used to describe the technical details of the analysis. The TYPE option is used to describe the type of analysis that

is to be performed. By selecting MIXTURE, a mixture model will be estimated.

When TYPE=MIXTURE is specified, either user-specified or automatic starting values are used to create randomly perturbed sets of starting values for all parameters in the model except variances and covariances. In this example, the random perturbations are based on automatic starting values. Maximum likelihood optimization is done in two stages. In the initial stage, 20 random sets of starting values are generated. An optimization is carried out for ten iterations using each of the 20 random sets of starting values. The ending values from the 4 optimizations with the highest loglikelihoods are used as the starting values in the final stage optimizations which are carried out using the default optimization settings for TYPE=MIXTURE. A more thorough investigation of multiple solutions can be carried out using the STARTS and STITERATIONS options of the ANALYSIS command.

```
MODEL:
            %OVERALL%
            y ON x1 x2;
            c ON x1;
            %c#2%
            y ON x2;
            y;
```

The MODEL command is used to describe the model to be estimated. For mixture models, there is an overall model designated by the label %OVERALL%. The overall model describes the part of the model that is in common for all latent classes. The part of the model that differs for each class is specified by a label that consists of the categorical latent variable followed by the number sign followed by the class number. In the example above, the label %c#2% refers to the part of the model for class 2 that differs from the overall model.

In the overall model, the first ON statement describes the linear regression of y on the covariates x1 and x2. The second ON statement describes the multinomial logistic regression of the categorical latent variable c on the covariate x1 when comparing class 1 to class 2. The intercept in the regression of c on x1 is estimated as the default.

In the model for class 2, the ON statement describes the linear regression of y on the covariate x2. This specification relaxes the default equality

constraint for the regression coefficient. By mentioning the residual variance of y, it is not held equal across classes. The intercepts in class 1 and class 2 are free and unequal as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator.

Following is an alternative specification of the multinomial logistic regression of c on the covariate x1:

c#1 ON x1;

where c#1 refers to the first class of c. The classes of a categorical latent variable are referred to by adding to the name of the categorical latent variable the number sign (#) followed by the number of the class. This alternative specification allows individual parameters to be referred to in the MODEL command for the purpose of giving starting values or placing restrictions.

```
OUTPUT:     TECH1 TECH8;
```

The OUTPUT command is used to request additional output not included as the default. The TECH1 option is used to request the arrays containing parameter specifications and starting values for all free parameters in the model. The TECH8 option is used to request that the optimization history in estimating the model be printed in the output. TECH8 is printed to the screen during the computations as the default. TECH8 screen printing is useful for determining how long the analysis takes.

EXAMPLE 7.2: MIXTURE REGRESSION ANALYSIS FOR A COUNT VARIABLE USING A ZERO-INFLATED POISSON MODEL USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a mixture regression
           analysis for a count variable using a
           zero-inflated Poisson model using
           automatic starting values with random
           starts
DATA:      FILE IS ex7.2.dat;
VARIABLE:  NAMES ARE u x1 x2;
           CLASSES = c (2);
           COUNT = u (i);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           u ON x1 x2;
           u#1 ON x1 x2;
           c ON x1;
           %c#2%
           u ON x2;
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.1 is that the dependent variable is a count variable instead of a continuous variable. The COUNT option is used to specify which dependent variables are treated as count variables in the model and its estimation and whether a Poisson or zero-inflated Poisson model will be estimated. In the example above, u is a count variable. The i in parentheses following u indicates that a zero-inflated Poisson model will be estimated.

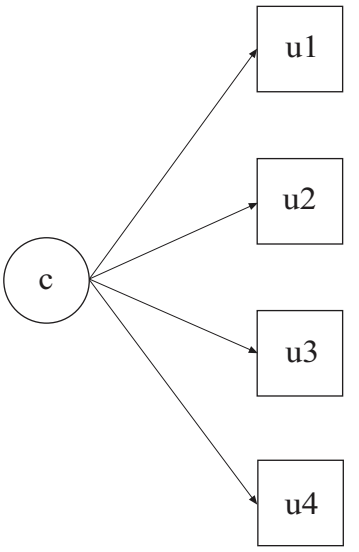
With a zero-inflated Poisson model, two regressions are estimated. In the overall model, the first ON statement describes the Poisson regression of the count part of u on the covariates x1 and x2. This regression predicts the value of the count dependent variable for individuals who are able to assume values of zero and above. The second ON statement describes the logistic regression of the binary latent inflation variable u#1 on the covariates x1 and x2. This regression describes the probability of being unable to assume any value except zero. The inflation variable is referred to by adding to the name of the count variable the number sign (#) followed by the number 1. The

third ON statement specifies the multinomial logistic regression of the categorical latent variable c on the covariate x1 when comparing class 1 to class 2. The intercept in the regression of c on x1 is estimated as the default.

In the model for class 2, the ON statement describes the Poisson regression of the count part of u on the covariate x2. This specification relaxes the default equality constraint for the regression coefficient. The intercepts of u are free and unequal across classes as the default. All other parameters are held equal across classes as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.3: LCA WITH BINARY LATENT CLASS INDICATORS USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a LCA with binary
           latent class indicators using automatic
           starting values with random starts
DATA:      FILE IS ex7.3.dat;
VARIABLE:  NAMES ARE u1-u4 x1-x10;
           USEVARIABLES = u1-u4;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
           AUXILIARY = x1-x10 (R3STEP);
ANALYSIS:  TYPE = MIXTURE;
OUTPUT:    TECH1 TECH8 TECH10;
```



In this example, the latent class analysis (LCA) model with binary latent class indicators shown in the picture above is estimated using automatic starting values and random starts. Because c is a categorical latent variable, the interpretation of the picture is not the same as for models with continuous latent variables. The arrows from c to the latent class indicators u1, u2, u3, and u4 indicate that the thresholds of the latent class indicators vary across the classes of c. This implies that the probabilities of the latent class indicators vary across the classes of c. The arrows correspond to the regressions of the latent class indicators on a set of dummy variables representing the categories of c.

The CATEGORICAL option is used to specify which dependent variables are treated as binary or ordered categorical (ordinal) variables in the model and its estimation. In the example above, the latent class indicators u1, u2, u3, and u4, are binary or ordered categorical variables. The program determines the number of categories for each indicator. The AUXILIARY option is used to specify variables that are not part of the analysis that are important predictors of latent classes using a three-step approach (Vermunt, 2010; Asparouhov & Muthén, 2012b). The letters R3STEP in parentheses is placed behind the variables in the AUXILIARY statement that that will be used as covariates in the third step multinomial logistic regression in a mixture model.

The MODEL command does not need to be specified when automatic starting values are used. The thresholds of the observed variables and the mean of the categorical latent variable are estimated as the default. The thresholds are not held equal across classes as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator.

The TECH10 option is used to request univariate, bivariate, and response pattern model fit information for the categorical dependent variables in the model. This includes observed and estimated (expected) frequencies and standardized residuals. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.4: LCA WITH BINARY LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITHOUT RANDOM STARTS

```
TITLE:     this is an example of a LCA with binary
           latent class indicators using user-
           specified starting values without random
           starts
DATA:      FILE IS ex7.4.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 0;
MODEL:
           %OVERALL%
           %c#1%
           [u1$1*1 u2$1*1 u3$1*-1 u4$1*-1];
           %c#2%
           [u1$1*-1 u2$1*-1 u3$1*1 u4$1*1];
OUTPUT:    TECH1 TECH8;
```

The differences between this example and Example 7.3 are that user-specified starting values are used instead of automatic starting values and there are no random starts. By specifying STARTS=0 in the ANALYSIS command, random starts are turned off.

In the MODEL command, user-specified starting values are given for the thresholds of the binary latent class indicators. For binary and ordered categorical dependent variables, thresholds are referred to by adding to a variable name a dollar sign ($) followed by a threshold number. The number of thresholds is equal to the number of categories minus one. Because the latent class indicators are binary, they have one threshold. The thresholds of the latent class indicators are referred to as u1$1, u2$1, u3$1, and u4$1. Square brackets are used to specify starting values in the logit scale for the thresholds of the binary latent class indicators. The asterisk (*) is used to assign a starting value. It is placed after a variable with the starting value following it. In the example above, the threshold of u1 is assigned the starting value of 1 for class 1 and -1 for class 2. The threshold of u4 is assigned the starting value of -1 for class 1 and 1 for class 2. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.3.

EXAMPLE 7.5: LCA WITH BINARY LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a LCA with binary
           latent class indicators using user-
           specified starting values with random
           starts
DATA:      FILE IS ex7.5.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 100 10;
           STITERATIONS = 20;
MODEL:
           %OVERALL%
           %c#1%
           [u1$1*1 u2$1*1 u3$1*-1 u4$1*-1];
           %c#2%
           [u1$1*-1 u2$1*-1 u3$1*1 u4$1*1];
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.4 is that random starts are used. In this example, the random perturbations are based on user-specified starting values. The STARTS option is used to specify the number of initial stage random sets of starting values to generate and the number of final stage optimizations to use. The default is 20 random sets of starting values for the initial stage and 4 optimizations for the final stage. In the example above, the STARTS option specifies that 100 random sets of starting values for the initial stage and 10 final stage optimizations will be used. The STITERATIONS option is used to specify the maximum number of iterations allowed in the initial stage. In this example, 20 iterations are allowed in the initial stage instead of the default of 10. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1, 7.3, and 7.4.

EXAMPLE 7.6: LCA WITH THREE-CATEGORY LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITHOUT RANDOM STARTS

```
TITLE:     this is an example of a LCA with three-
           category latent class indicators using
           user-specified starting values without
           random starts
DATA:      FILE IS ex7.6.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 0;
MODEL:
           %OVERALL%
           %c#1%
           [u1$1*.5 u2$1*.5 u3$1*-.5 u4$1*-.5];
           [u1$2*1 u2$2*1 u3$2*0 u4$2*0];
           %c#2%
           [u1$1*-.5 u2$1*-.5 u3$1*.5 u4$1*.5];
           [u1$2*0 u2$2*0 u3$2*1 u4$2*1];
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.4 is that the latent class indicators are ordered categorical (ordinal) variables with three categories instead of binary variables. When latent class indicators are ordered categorical variables, each latent class indicator has more than one threshold. The number of thresholds is equal to the number of categories minus one. When user-specified starting values are used, they must be specified for all thresholds and they must be in increasing order for each variable within each class. For example, in class 1 the threshold starting values for latent class indicator u1 are .5 for the first threshold and 1 for the second threshold. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1, 7.3, and 7.4.

EXAMPLE 7.7: LCA WITH UNORDERED CATEGORICAL LATENT CLASS INDICATORS USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a LCA with unordered
           categorical latent class indicators using
           automatic starting values with random
           starts
DATA:      FILE IS ex7.7.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           NOMINAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.3 is that the latent class indicators are unordered categorical (nominal) variables instead of binary variables. The NOMINAL option is used to specify which dependent variables are treated as unordered categorical (nominal) variables in the model and its estimation. In the example above, u1, u2, u3, and u4 are three-category unordered variables. The categories of an unordered categorical variable are referred to by adding to the name of the unordered categorical variable the number sign (#) followed by the number of the category. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different

estimator. An explanation of the other commands can be found in Examples 7.1 and 7.3.

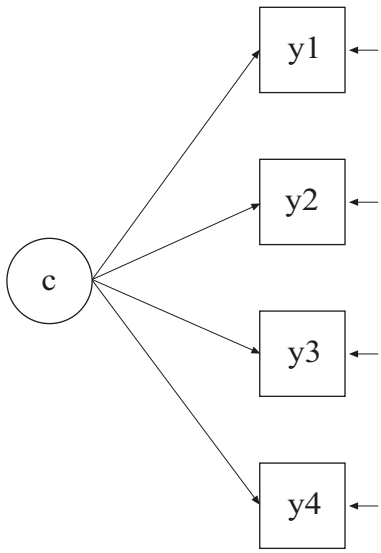
EXAMPLE 7.8: LCA WITH UNORDERED CATEGORICAL LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a LCA with unordered
           categorical latent class indicators using
           user-specified starting values with random
           starts
DATA:      FILE IS ex7.8.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           NOMINAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
MODEL:     %OVERALL%
           %c#1%
           [u1#1-u4#1*0];
           [u1#2-u4#2*1];
           %c#2%
           [u1#1-u4#1*-1];
           [u1#2-u4#2*-1];
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.7 is that user-specified starting values are used instead of automatic starting values. Means are referred to by using bracket statements. The categories of an unordered categorical variable are referred to by adding to the name of the unordered categorical variable the number sign (#) followed by the number of the category. In this example, u1#1 refers to the first category of u1 and u1#2 refers to the second category of u1. Starting values of 0 and 1 are given for the means in class 1 and starting values of -1 are given for the means in class 2. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1, 7.3, and 7.7.

EXAMPLE 7.9: LCA WITH CONTINUOUS LATENT CLASS INDICATORS USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS

```
TITLE:     this is an example of a LCA with
           continuous latent class indicators using
           automatic starting values with random
           starts
DATA:      FILE IS ex7.9.dat;
VARIABLE:  NAMES ARE y1-y4;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
OUTPUT:    TECH1 TECH8;
```



The difference between this example and Example 7.3 is that the latent class indicators are continuous variables instead of binary variables. When there is no specification in the VARIABLE command regarding the scale of the dependent variables, it is assumed that they are continuous. Latent class analysis with continuous latent class indicators is often referred to as latent profile analysis.

The MODEL command does not need to be specified when automatic starting values are used. The means and variances of the latent class indicators and the mean of the categorical latent variable are estimated as the default. The means of the latent class indicators are not held equal across classes as the default. The variances are held equal across classes as the default and the covariances among the latent class indicators are fixed at zero as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.3.

EXAMPLE 7.10: LCA WITH CONTINUOUS LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITHOUT RANDOM STARTS

```
TITLE:     this is an example of a LCA with
           continuous latent class indicators using
           user-specified starting values without
           random starts
DATA:      FILE IS ex7.10.dat;
VARIABLE:  NAMES ARE y1-y4;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 0;
MODEL:
           %OVERALL%
           %c#1%
           [y1–y4*1];
           y1-y4;
           %c#2%
           [y1–y4*-1];
           y1-y4;
OUTPUT:    TECH1 TECH8;
```

The difference between this example and Example 7.4 is that the latent class indicators are continuous variables instead of binary variables. As a result, starting values are given for means instead of thresholds.

The means and variances of the latent class indicators and the mean of the categorical latent variable are estimated as the default. In the models for class 1 and class 2, by mentioning the variances of the latent class

indicators, the default constraint of equality of variances across classes is relaxed. The covariances among the latent class indicators within class are fixed at zero as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.4.

EXAMPLE 7.11: LCA WITH BINARY, CENSORED, UNORDERED, AND COUNT LATENT CLASS INDICATORS USING USER-SPECIFIED STARTING VALUES WITHOUT RANDOM STARTS

```
TITLE:     this is an example of a LCA with binary,
           censored, unordered, and count latent
           class indicators using user-specified
           starting values without random starts
DATA:      FILE IS ex7.11.dat;
VARIABLE:  NAMES ARE u1 y1 u2 u3;
           CLASSES = c (2);
           CATEGORICAL = u1;
           CENSORED = y1 (b);
           NOMINAL = u2;
           COUNT = u3 (i);
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 0;
MODEL:
           %OVERALL%
           %c#1%
           [u1$1*-1 y1*3 u2#1*0 u2#2*1 u3*.5
           u3#1*1.5];
           y1*2;
           %c#2%
           [u1$1*0 y1*1 u2#1*-1 u2#2*0 u3*1 u3#1*1];
           y1*1;
OUTPUT:    TECH1 TECH8;
```

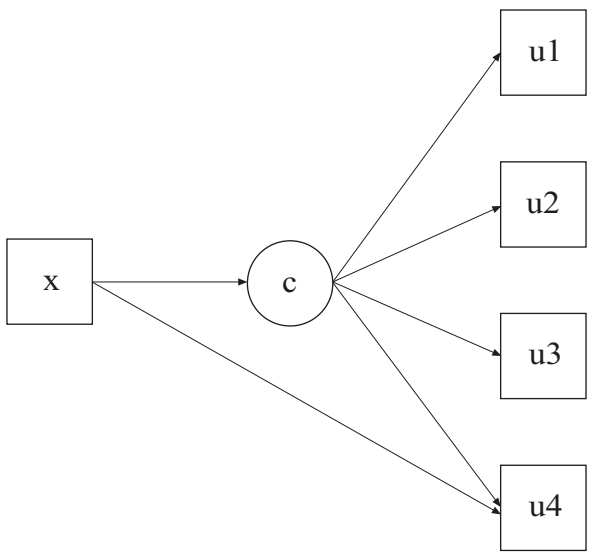
The difference between this example and Example 7.4 is that the latent class indicators are a combination of binary, censored, unordered categorical (nominal) and count variables instead of binary variables.

The CATEGORICAL option is used to specify which dependent variables are treated as binary or ordered categorical (ordinal) variables

in the model and its estimation. In the example above, the latent class indicator u1 is a binary variable. The CENSORED option is used to specify which dependent variables are treated as censored variables in the model and its estimation, whether they are censored from above or below, and whether a censored or censored-inflated model will be estimated. In the example above, y1 is a censored variable. The b in parentheses following y1 indicates that y1 is censored from below, that is, has a floor effect, and that the model is a censored regression model. The censoring limit is determined from the data. The NOMINAL option is used to specify which dependent variables are treated as unordered categorical (nominal) variables in the model and its estimation. In the example above, u2 is a three-category unordered variable. The program determines the number of categories. The categories of an unordered categorical variable are referred to by adding to the name of the unordered categorical variable the number sign (#) followed by the number of the category. In this example, u2#1 refers to the first category of u2 and u2#2 refers to the second category of u2. The COUNT option is used to specify which dependent variables are treated as count variables in the model and its estimation and whether a Poisson or zero-inflated Poisson model will be estimated. In the example above, u3 is a count variable. The i in parentheses following u3 indicates that a zero-inflated model will be estimated. The inflation part of the count variable is referred to by adding to the name of the count variable the number sign (#) followed by the number 1. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.4.

EXAMPLE 7.12: LCA WITH BINARY LATENT CLASS INDICATORS USING AUTOMATIC STARTING VALUES WITH RANDOM STARTS WITH A COVARIATE AND A DIRECT EFFECT

```
TITLE:     this is an example of a LCA with binary
           latent class indicators using automatic
           starting values with random starts with a
           covariate and a direct effect
DATA:      FILE IS ex7.12.dat;
VARIABLE:  NAMES ARE u1-u4 x;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           c ON x;
           u4 ON x;
OUTPUT:    TECH1 TECH8;
```



The difference between this example and Example 7.3 is that the model contains a covariate and a direct effect. The first ON statement

describes the multinomial logistic regression of the categorical latent variable c on the covariate x when comparing class 1 to class 2. The intercepts of this regression are estimated as the default. The second ON statement describes the logistic regression of the binary indicator u4 on the covariate x. This is referred to as a direct effect from x to u4. The regression coefficient is held equal across classes as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.3.

EXAMPLE 7.13: CONFIRMATORY LCA WITH BINARY LATENT CLASS INDICATORS AND PARAMETER CONSTRAINTS

```
TITLE:     this is an example of a confirmatory LCA
           with binary latent class indicators and
           parameter constraints
DATA:      FILE IS ex7.13.dat;
VARIABLE:  NAMES ARE u1-u4;
           CLASSES = c (2);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           %c#1%
           [u1$1*-1];
           [u2$1-u3$1*-1] (1);
           [u4$1*-1] (p1);
           %c#2%
           [u1$1@-15];
           [u2$1-u3$1*1] (2);
           [u4$1*1] (p2);
MODEL CONSTRAINT:
           p2 = - p1;
OUTPUT:    TECH1 TECH8;
```

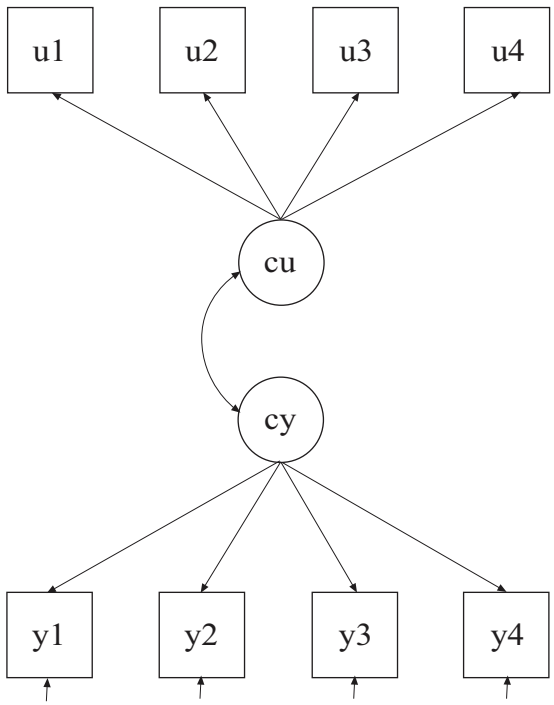
In this example, constraints are placed on the measurement parameters of the latent class indicators to reflect three hypotheses: (1) u2 and u3 are parallel measurements, (2) u1 has a probability of one in class 2, and (3) the error rate for u4 is the same in the two classes (McCutcheon, 2002, pp. 70-72).

The first hypothesis is specified by placing (1) following the threshold parameters for u2 and u3 in class 1 and (2) following the threshold parameters for u2 and u3 in class 2. This holds the thresholds for the two latent class indicators equal to each other but not equal across classes. The second hypothesis is specified by fixing the threshold of u1 in class 2 to the logit value of -15. The third hypothesis is specified using the MODEL CONSTRAINT command. The MODEL CONSTRAINT command is used to define linear and non-linear constraints on the parameters in the model. Parameters are given labels by placing a name in parentheses after the parameter in the MODEL command. In the MODEL command, the threshold of u4 in class 1 is given the label p1 and the threshold of u4 in class 2 is given the label p2. In the MODEL CONSTRAINT command, the linear constraint is defined. The threshold of u4 in class 1 is equal to the negative value of the threshold of u4 in class 2. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.14: CONFIRMATORY LCA WITH TWO CATEGORICAL LATENT VARIABLES

```
TITLE:     this is an example of a confirmatory LCA
           with two categorical latent variables
DATA:      FILE IS ex7.14.dat;
VARIABLE:  NAMES ARE u1-u4 y1-y4;
           CLASSES = cu (2) cy (3);
           CATEGORICAL = u1-u4;
ANALYSIS:  TYPE = MIXTURE;
           PARAMETERIZATION = LOGLINEAR;
MODEL:
           %OVERALL%
           cu WITH cy;
```

```
MODEL cu:
            %cu#1%
            [u1$1-u4$1];
            %cu#2%
            [u1$1-u4$1];
MODEL cy:
            %cy#1%
            [y1-y4];
            %cy#2%
            [y1-y4];
            %cy#3%
            [y1-y4];
OUTPUT:     TECH1 TECH8;
```



In this example, the confirmatory LCA with two categorical latent variables shown in the picture above is estimated. The two categorical latent variables are correlated and have their own sets of latent class indicators.

The CLASSES option is used to assign names to the categorical latent variables in the model and to specify the number of latent classes in the model for each categorical latent variable. In the example above, there are two categorical latent variables cu and cy. The categorical latent variable cu has two latent classes and the categorical latent variable cy has three latent classes. PARAMETERIZATION=LOGLINEAR is used to specify associations among categorical latent variables. In the LOGLINEAR parameterization, the WITH option of the MODEL command is used to specify the relationships between the categorical latent variables. When a model has more than one categorical latent variable, MODEL followed by a label is used to describe the analysis model for each categorical latent variable. Labels are defined by using the names of the categorical latent variables. The categorical latent variable cu has four binary indicators u1 through u4. Their thresholds are specified to vary only across the classes of the categorical latent variable cu. The categorical latent variable cy has four continuous indicators y1 through y4. Their means are specified to vary only across the classes of the categorical latent variable cy. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

Following is an alternative specification of the associations among cu and cy:

cu#1 WITH cy#1 cy#2;

where cu#1 refers to the first class of cu, cy#1 refers to the first class of cy, and cy#2 refers to the second class of cy. The classes of a categorical latent variable are referred to by adding to the name of the categorical latent variable the number sign (#) followed by the number of the class. This alternative specification allows individual parameters to be referred to in the MODEL command for the purpose of giving starting values or placing restrictions.

EXAMPLE 7.15: LOGLINEAR MODEL FOR A THREE-WAY TABLE WITH CONDITIONAL INDEPENDENCE BETWEEN THE FIRST TWO VARIABLES

```
TITLE:     this is an example of a loglinear model
           for a three-way table with conditional
           independence between the first two
           variables
DATA:      FILE IS ex7.15.dat;
VARIABLE:  NAMES ARE u1 u2 u3 w;
           FREQWEIGHT = w;
           CATEGORICAL = u1-u3;
           CLASSES = c1 (2) c2 (2) c3 (2);
ANALYSIS:  TYPE = MIXTURE;
           STARTS = 0;
           PARAMETERIZATION = LOGLINEAR;
MODEL:
           %OVERALL%
           c1 WITH c3;
           c2 WITH c3;
MODEL c1:
           %c1#1%
           [u1$1@15];
           %c1#2%
           [u1$1@-15];
MODEL c2:
           %c2#1%
           [u2$1@15];
           %c2#2%
           [u2$1@-15];
MODEL c3:
           %c3#1%
           [u3$1@15];
           %c3#2%
           [u3$1@-15];
OUTPUT:    TECH1 TECH8;
```

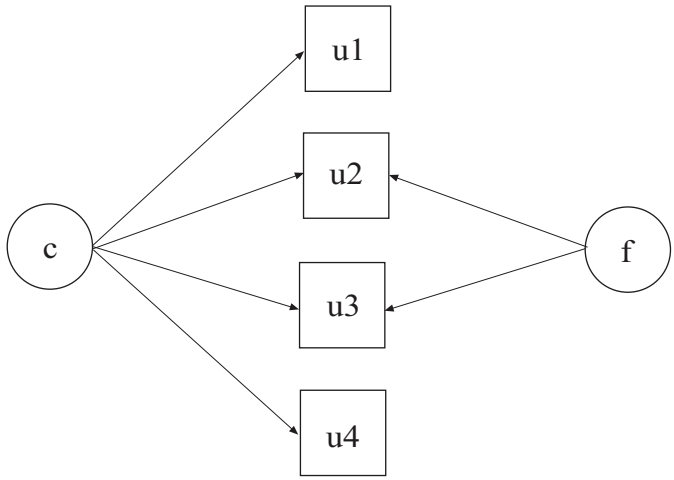
In this example, a loglinear model for a three-way frequency table with conditional independence between the first two variables is estimated. The loglinear model is estimated using categorical latent variables that are perfectly measured by observed categorical variables. It is also possible to estimate loglinear models for categorical latent variables that are measured with error by observed categorical variables. The conditional independence is specified by the two-way interaction

between the first two variables being zero for each of the two levels of the third variable.

PARAMETERIZATION=LOGLINEAR is used to estimate loglinear models with two- and three-way interactions. In the LOGLINEAR parameterization, the WITH option of the MODEL command is used to specify the associations among the categorical latent variables. When a model has more than one categorical latent variable, MODEL followed by a label is used to describe the analysis model for each categorical latent variable. Labels are defined by using the names of the categorical latent variables. In the example above, the categorical latent variables are perfectly measured by the latent class indicators. This is specified by fixing their thresholds to the logit value of plus or minus 15, corresponding to probabilities of zero and one. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.14.

EXAMPLE 7.16: LCA WITH PARTIAL CONDITIONAL INDEPENDENCE

```
TITLE:     this is an example of LCA with partial
           conditional independence
DATA:      FILE IS ex7.16.dat;
VARIABLE:  NAMES ARE u1-u4;
           CATEGORICAL = u1-u4;
           CLASSES = c(2);
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
MODEL:
           %OVERALL%
           f by u2-u3@0;
           f@1; [f@0];
           %c#1%
           [u1$1-u4$1*-1];
           f by u2@1 u3;
OUTPUT:    TECH1 TECH8;
```

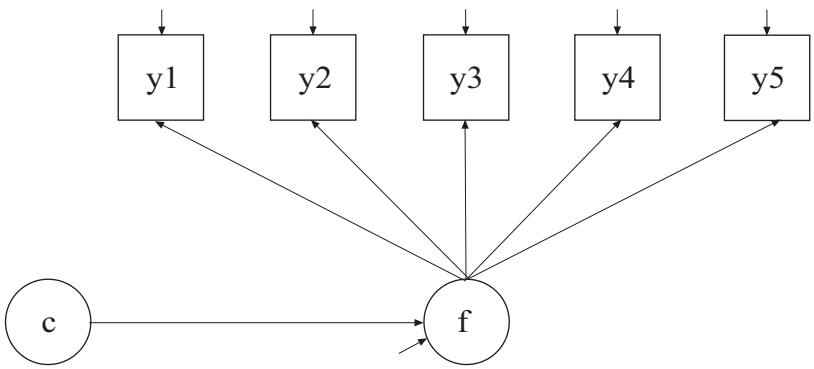


In this example, the LCA with partial conditional independence shown in the picture above is estimated. A similar model is described in Qu, Tan, and Kutner (1996).

By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, one dimension of integration is used with 15 integration points. The ESTIMATOR option can be used to select a different estimator. In the example above, the lack of conditional independence between the latent class indicators u2 and u3 in class 1 is captured by u2 and u3 being influenced by the continuous latent variable f in class 1. The conditional independence assumption for u2 and u3 is not violated for class 2. This is specified by fixing the factor loadings to zero in the overall model. The amount of deviation from conditional independence between u2 and u3 in class 1 is captured by the u3 factor loading for the continuous latent variable f. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.17: MIXTURE CFA MODELING

```
TITLE:     this is an example of mixture CFA modeling
DATA:      FILE IS ex7.17.dat;
VARIABLE:  NAMES ARE y1-y5;
           CLASSES = c(2);
ANALYSIS:  TYPE = MIXTURE;
MODEL:     %OVERALL%
           f BY y1-y5;
           %c#1%
           [f*1];
OUTPUT:    TECH1 TECH8;
```

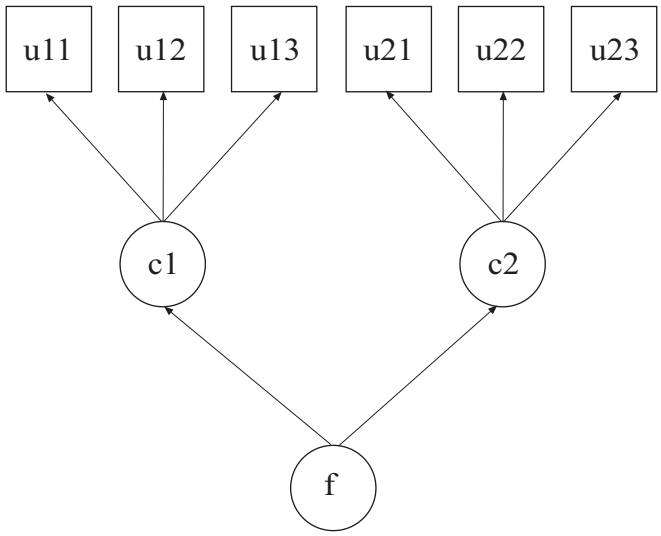


In this example, the mixture CFA model shown in the picture above is estimated (Muthén, 2008). The mean of the factor f varies across the classes of the categorical latent variable c. The residual arrow pointing to f indicates that the factor varies within class. This implies that the distribution of f is allowed to be non-normal. It is possible to allow other parameters of the CFA model to vary across classes.

The BY statement specifies that f is measured by y1, y2, y3, y4, and y5. The factor mean varies across the classes. All other model parameters are held equal across classes as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.18: LCA WITH A SECOND-ORDER FACTOR (TWIN ANALYSIS)

```
TITLE:     this is an example of a LCA with a second-
           order factor (twin analysis)
DATA:      FILE IS ex7.18.dat;
VARIABLE:  NAMES ARE u11-u13 u21-u23;
           CLASSES = c1(2) c2(2);
           CATEGORICAL = u11-u23;
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
MODEL:
           %OVERALL%
           f BY;
           f@1;
           c1 c2 ON f*1 (1);
MODEL c1:
           %c1#1%
           [u11$1-u13$1*-1];
           %c1#2%
           [u11$1-u13$1*1];
MODEL c2:
           %c2#1%
           [u21$1-u23$1*-1];
           %c2#2%
           [u21$1-u23$1*1];
OUTPUT:    TECH1 TECH8;
```



In this example, the second-order factor model shown in the picture above is estimated. The first-order factors are categorical latent variables and the second-order factor is a continuous latent variable. This is a model that can be used for studies of twin associations where the categorical latent variable c1 refers to twin 1 and the categorical latent variable c2 refers to twin 2.

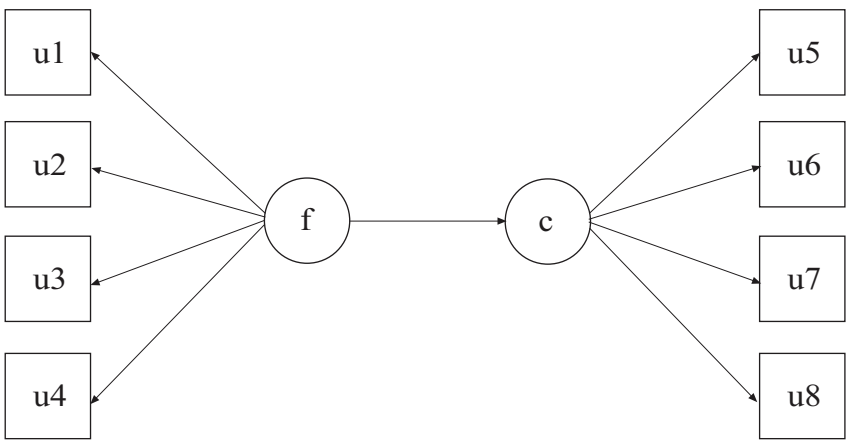
By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, one dimension of integration is used with 15 integration points. The ESTIMATOR option can be used to select a different estimator. When a model has more than one categorical latent variable, MODEL followed by a label is used to describe the analysis model for each categorical latent variable. Labels are defined by using the names of the categorical latent variables.

In the overall model, the BY statement names the second order factor f. The ON statement specifies that f influences both categorical latent variables in the same amount by imposing an equality constraint on the two multinomial logistic regression coefficients. The slope in the multinomial regression of c on f reflects the strength of association

between the two categorical latent variables. An explanation of the other commands can be found in Examples 7.1 and 7.14.

EXAMPLE 7.19: SEM WITH A CATEGORICAL LATENT VARIABLE REGRESSED ON A CONTINUOUS LATENT VARIABLE

```
TITLE:     this is an example of a SEM with a
           categorical latent variable regressed on a
           continuous latent variable
DATA:      FILE IS ex7.19.dat;
VARIABLE:  NAMES ARE u1-u8;
           CATEGORICAL = u1-u8;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
MODEL:
           %OVERALL%
           f BY u1-u4;
           c ON f;
           %c#1%
           [u5$1-u8$1];
           %c#2%
           [u5$1-u8$1];
OUTPUT:    TECH1 TECH8;
```

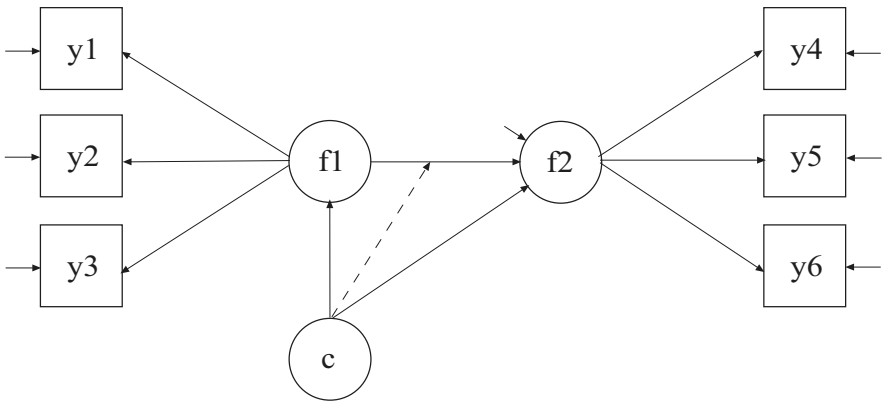


In this example, the model with both a continuous and categorical latent variable shown in the picture above is estimated. The categorical latent variable c is regressed on the continuous latent variable f in a multinomial logistic regression.

By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, one dimension of integration is used with 15 integration points. The ESTIMATOR option can be used to select a different estimator. In the overall model, the BY statement specifies that f is measured by the categorical factor indicators u1 through u4. The categorical latent variable c has four binary latent class indicators u5 through u8. The ON statement specifies the multinomial logistic regression of the categorical latent variable c on the continuous latent variable f. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.20: STRUCTURAL EQUATION MIXTURE MODELING

```
TITLE:     this is an example of structural equation
           mixture modeling
DATA:      FILE IS ex7.20.dat;
VARIABLE:  NAMES ARE y1-y6;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           f1 BY y1-y3;
           f2 BY y4-y6;
           f2 ON f1;
           %c#1%
           [f1*1 f2];
           f2 ON f1;
OUTPUT:    TECH1 TECH8;
```

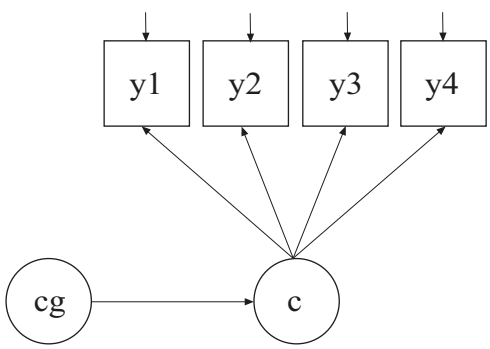


In this example, the structural equation mixture model shown in the picture above is estimated. A continuous latent variable f2 is regressed on a second continuous latent variable f1. The solid arrows from the categorical latent variable c to f1 and f2 indicate that the mean of f1 and the intercept of f2 vary across classes. The broken arrow from c to the arrow from f1 to f2 indicates that the slope in the linear regression of f2 on f1 varies across classes. For related models, see Jedidi, Jagpal, and DeSarbo (1997).

In the overall model, the first BY statement specifies that f1 is measured by y1 through y3. The second BY statement specifies that f2 is measured by y4 through y6. The ON statement describes the linear regression of f2 on f1. In the model for class 1, the mean of f1, the intercept of f2, and the slope in the regression of f2 on f1 are specified to be free across classes. All other parameters are held equal across classes as the default. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.21: MIXTURE MODELING WITH KNOWN CLASSES (MULTIPLE GROUP ANALYSIS)

```
TITLE:     this is an example of mixture modeling
           with known classes (multiple group
           analysis)
DATA:      FILE IS ex7.21.dat;
VARIABLE:  NAMES = g y1-y4;
           CLASSES = cg (2) c (2);
           KNOWNCLASS = cg (g = 0 g = 1);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           c ON cg;
MODEL c:
           %c#1%
           [y1-y4];
           %c#2%
           [y1-y4];
MODEL cg:
           %cg#1%
           y1-y4;
           %cg#2%
           y1-y4;
OUTPUT:    TECH1 TECH8;
```

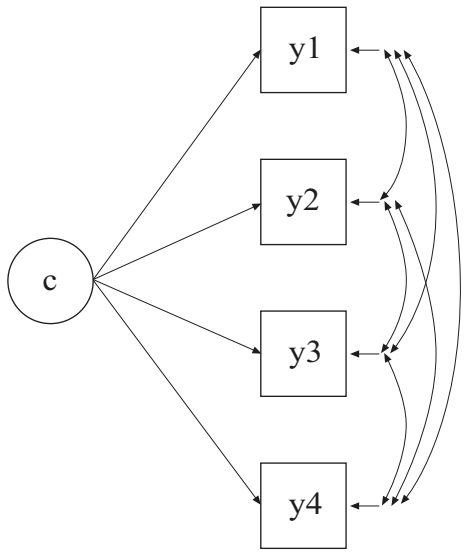


In this example, the multiple group mixture model shown in the picture above is estimated. The groups are represented by the classes of the categorical latent variable cg, which has known class (group) membership.

The KNOWNCLASS option is used for multiple group analysis with TYPE=MIXTURE. It is used to identify the categorical latent variable for which latent class membership is known and is equal to observed groups in the sample. The KNOWNCLASS option identifies cg as the categorical latent variable for which latent class membership is known. The information in parentheses following the categorical latent variable name defines the known classes using an observed variable. In this example, the observed variable g is used to define the known classes. The first class consists of individuals with the value 0 on the variable g. The second class consists of individuals with the value 1 on the variable g. The means of y1, y2, y3, and y4 vary across the classes of c, while the variances of y1, y2, y3, and y4 vary across the classes of cg. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.22: MIXTURE MODELING WITH CONTINUOUS VARIABLES THAT CORRELATE WITHIN CLASS (MULTIVARIATE NORMAL MIXTURE MODEL)

```
TITLE:     this is an example of mixture modeling
           with continuous variables that correlate
           within class (multivariate normal mixture
           model)
DATA:      FILE IS ex7.22.dat;
VARIABLE:  NAMES ARE y1-y4;
           CLASSES = c (3);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           y1 WITH y2-y4;
           y2 WITH y3 y4;
           y3 WITH y4;
           %c#2%
           [y1-y4*-1];
           %c#3%
           [y1-y4*1];
OUTPUT:    TECH1 TECH8;
```



In this example, the mixture model shown in the picture above is estimated. Because c is a categorical latent variable, the interpretation of the picture is not the same as for models with continuous latent variables. The arrows from c to the observed variables y1, y2, y3, and y4 indicate that the means of the observed variables vary across the classes of c. The arrows correspond to the regressions of the observed variables on a set of dummy variables representing the categories of c. The observed variables correlate within class. This is a conventional multivariate mixture model (Everitt & Hand, 1981; McLachlan & Peel, 2000).

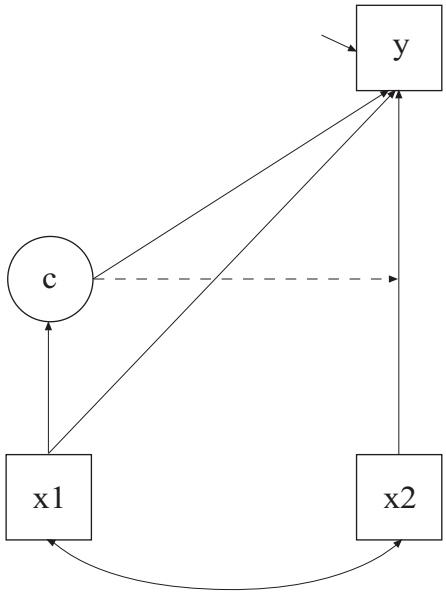
In the overall model, by specifying the three WITH statements the default of zero covariances within class is relaxed and the covariances among y1, y2, y3, and y4 are estimated. These covariances are held equal across classes as the default. The variances of y1, y2, y3, and y4 are estimated and held equal as the default. These defaults can be overridden. The means of the categorical latent variable c are estimated as the default.

When WITH statements are included in a mixture model, starting values may be useful. In the class-specific model for class 2, starting values of -1 are given for the means of y1, y2, y3, and y4. In the class-specific model for class 3, starting values of 1 are given for the means of y1, y2,

y3, and y4. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.23: MIXTURE RANDOMIZED TRIALS MODELING USING CACE ESTIMATION WITH TRAINING DATA

```
TITLE:     this is an example of mixture randomized
           trials modeling using CACE estimation with
           training data
DATA:      FILE IS ex7.23.dat;
VARIABLE:  NAMES ARE y x1 x2 c1 c2;
           CLASSES = c (2);
           TRAINING = c1 c2;
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           y ON x1 x2;
           c ON x1;
           %c#1%
           [y];
           y;
           y ON x2@0;
           %c#2%
           [y*.5];
           y;
OUTPUT:    TECH1 TECH8;
```



In this example, the mixture model for randomized trials using CACE (Complier-Average Causal Effect) estimation with training data shown in the picture above is estimated (Little & Yau, 1998). The continuous dependent variable y is regressed on the covariate x1 and the treatment dummy variable x2. The categorical latent variable c is compliance status, with class 1 referring to non-compliers and class 2 referring to compliers. Compliance status is observed in the treatment group and unobserved in the control group. Because c is a categorical latent variable, the interpretation of the picture is not the same as for models with continuous latent variables. The arrow from c to the y variable indicates that the intercept of y varies across the classes of c. The arrow from c to the arrow from x2 to y indicates that the slope in the regression of y on x2 varies across the classes of c. The arrow from x1 to c represents the multinomial logistic regression of c on x1.

The TRAINING option is used to identify the variables that contain information about latent class membership. Because there are two classes, there are two training variables c1 and c2. Individuals in the treatment group are assigned values of 1 for c1 and 0 for c2 if they are non-compliers and 0 for c1 and 1 for c2 if they are compliers. Individuals in the control group are assigned values of 1 for both c1 and

c2 to indicate that they are allowed to be a member of either class and that their class membership is estimated.

In the overall model, the first ON statement describes the linear regression of y on the covariate x1 and the treatment dummy variable x2. The intercept and residual variance of y are estimated as the default. The second ON statement describes the multinomial logistic regression of the categorical latent variable c on the covariate x1 when comparing class 1 to class 2. The intercept in the regression of c on x1 is estimated as the default.

In the model for class 1, a starting value of zero is given for the intercept of y as the default. The residual variance of y is specified to relax the default across class equality constraint. The ON statement describes the linear regression of y on x2 where the slope is fixed at zero. This is done because non-compliers do not receive treatment. In the model for class 2, a starting value of .5 is given for the intercept of y. The residual variance of y is specified to relax the default across class equality constraint. The regression of y ON x2, which represents the CACE treatment effect, is not fixed at zero for class 2. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

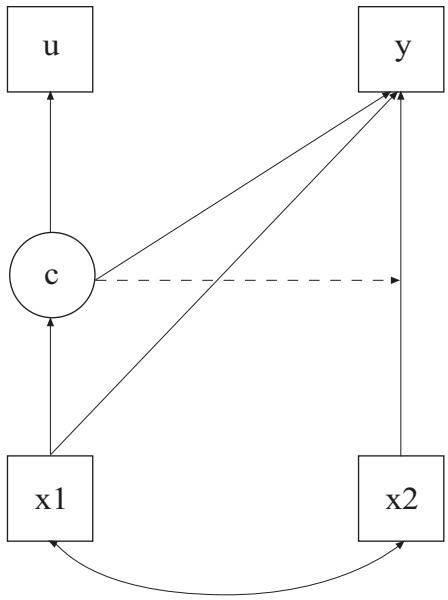
EXAMPLE 7.24: MIXTURE RANDOMIZED TRIALS MODELING USING CACE ESTIMATION WITH MISSING DATA ON THE LATENT CLASS INDICATOR

```
TITLE:     this is an example of mixture randomized
           trials modeling using CACE estimation with
           missing data on the latent class indicator
DATA:      FILE IS ex7.24.dat;
VARIABLE:  NAMES ARE u y x1 x2;
           CLASSES = c (2);
           CATEGORICAL = u;
           MISSING = u (999);
ANALYSIS:  TYPE = MIXTURE;
```

```
MODEL:
            %OVERALL%
            y ON x1 x2;
            c ON x1;

            %c#1%
            [u$1@15];
            [y];
            y;
            y ON x2@0;

            %c#2%
            [u$1@-15];
            [y*.5];
            y;
OUTPUT:     TECH1 TECH8;
```

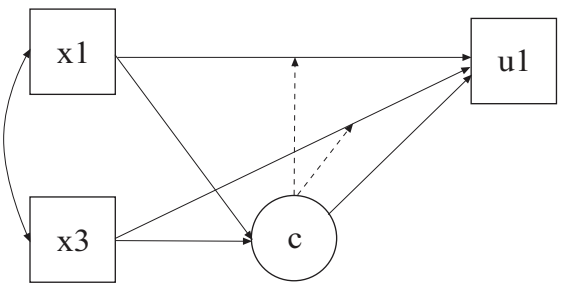


The difference between this example and Example 7.23 is that a binary latent class indicator u has been added to the model. This binary variable represents observed compliance status. Treatment compliers have a value of 1 on this variable; treatment non-compliers have a value of 0 on this variable; and individuals in the control group have a missing value on this variable. The latent class indicator u is used instead of training data.

In the model for class 1, the threshold of the latent class indicator variable u is set to a logit value of 15. In the model for class 2, the threshold of the latent class indicator variable u is set to a logit value of –15. These logit values reflect that c is perfectly measured by u. Individuals in the non-complier class (class 1) have probability zero of observed compliance and individuals in the complier class (class 2) have probability one of observed compliance. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Examples 7.1 and 7.23.

EXAMPLE 7.25: ZERO-INFLATED POISSON REGRESSION CARRIED OUT AS A TWO-CLASS MODEL

```
TITLE:     this is an example of a zero-inflated
           Poisson regression carried out as a two-
           class model
DATA:      FILE IS ex3.8.dat;
VARIABLE:  NAMES ARE u1 x1 x3;
           COUNT IS u1;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
MODEL:
           %OVERALL%
           u1 ON x1 x3;
           c ON x1 x3;
           %c#1%
           [u1@-15];
           u1 ON x1@0 x3@0;
OUTPUT:    TECH1 TECH8;
```



In this example, the zero-inflated Poisson regression model shown in the picture above is estimated. This is an alternative to the way zero-inflated Poisson regression was carried out in Example 3.8. In the example above, a categorical latent variable c with two classes is used to represent individuals who are able to assume values of zero and above and individuals who are unable to assume any value except zero. The categorical latent variable c corresponds to the binary latent inflation variable u1#1 in Example 3.8. This approach has the advantage of allowing the estimation of the probability of being in each class and the posterior probabilities of being in each class for each individual.

The COUNT option is used to specify which dependent variables are treated as count variables in the model and its estimation and whether a Poisson or zero-inflated Poisson model will be estimated. In the example above, u1 is a specified as count variable without inflation because the inflation is captured by the categorical latent variable c.

In the overall model, the first ON statement describes the Poisson regression of the count variable u1 on the covariates x1 and x3. The second ON statement describes the multinomial logistic regression of the categorical latent variable c on the covariates x1 and x3 when comparing class 1 to class 2. In this example, class 1 contains individuals who are unable to assume any value except zero on u1. Class 2 contains individuals whose values on u1 are distributed as a Poisson variable without inflation. Mixing the two classes results in u1 having a zero-inflated Poisson distribution. In the class-specific model for class 1, the intercept of u1 is fixed at -15 to represent a low log rate at which the probability of a count greater than zero is zero. Therefore, all individuals in class 1 have a value of 0 on u1. Because u1 has no variability, the slopes in the Poisson regression of u1 on the covariates x1 and x3 in class 1 are fixed at zero. The default estimator for this type of analysis is maximum likelihood with robust standard errors. The ESTIMATOR option of the ANALYSIS command can be used to select a different estimator. An explanation of the other commands can be found in Example 7.1.

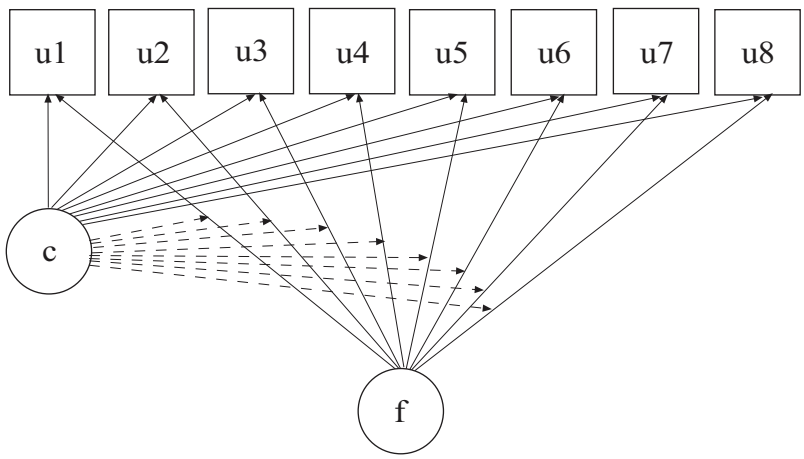
EXAMPLE 7.26: CFA WITH A NON-PARAMETRIC REPRESENTATION OF A NON-NORMAL FACTOR DISTRIBUTION

```
TITLE:     this is an example of CFA with a non-
           parametric representation of a non-normal
           factor distribution
DATA:      FILE IS ex7.26.dat;
VARIABLE:  NAMES ARE y1-y5 c;
           USEV = y1-y5;
           CLASSES = c (3);
ANALYSIS:  TYPE = MIXTURE;
MODEL:     %OVERALL%
           f BY y1-y5;
           f@0;
OUTPUT:    TECH1 TECH8;
```

In this example, a CFA model with a non-parametric representation of a non-normal factor distribution is estimated. One difference between this example and Example 7.17 is that the factor variance is fixed at zero in each class. This is done to capture a non-parametric representation of the factor distribution (Aitkin, 1999) where the latent classes are used to represent non-normality not unobserved heterogeneity with substantively meaningful latent classes. This is also referred to as semiparametric modeling. The factor distribution is represented by a histogram with as many bars as there are classes. The bars represent scale steps on the continuous latent variable. The spacing of the scale steps is obtained by the factor means in the different classes with a factor mean for one class fixed at zero for identification, and the percentage of individuals at the different scale steps is obtained by the latent class percentages. This means that continuous factor scores are obtained for the individuals while not assuming normality for the factor but estimating its distribution. Factor variances can also be estimated to obtain a more general mixture although this reverts to the parametric assumption of normality, in this case, within each class. When the latent classes are used to represent non-normality, the mixed parameter values are of greater interest than the parameters for each mixture component (Muthén, 2002, p. 102; Muthén, 2004). An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.27: FACTOR (IRT) MIXTURE ANALYSIS WITH BINARY LATENT CLASS AND FACTOR INDICATORS

```
TITLE:     this is an example of a factor (IRT)
           mixture analysis with binary latent class
           and factor indicators
DATA:      FILE = ex7.27.dat;
VARIABLE:  NAMES = u1-u8;
           CATEGORICAL = u1-u8;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
           STARTS = 100 20;
MODEL:     %OVERALL%
           f BY u1-u8;
           [f@0];
           %c#1%
           f BY u1@1 u2-u8;
           f;
           [u1$1-u8$1];
           %c#2%
           f BY u1@1 u2-u8;
           f;
           [u1$1-u8$1];
OUTPUT:    TECH1 TECH8;
```



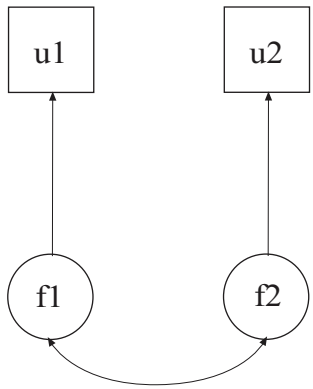
In this example, the factor (IRT) mixture model shown in the picture above is estimated (Muthén, 2008). The model is a generalization of the latent class model where the latent class model assumption of conditional independence between the latent class indicators within class is relaxed using a factor that influences the items within each class (Muthén, 2006; Muthén & Asparouhov, 2006; Muthén, Asparouhov, & Rebollo, 2006). The factor represents individual variation in response probabilities within class. Alternatively, this model may be seen as an Item Response Theory (IRT) mixture model. The broken arrows from the categorical latent variable c to the arrows from the factor f to the latent class indicators u1 to u8 indicate that the factor loadings vary across classes.

By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, one dimension of integration is used with 15 integration points. The ESTIMATOR option can be used to select a different estimator. The STARTS option is used to specify the number of initial stage random sets of starting values to generate and the number of final stage optimizations to use. The default is 20 random sets of starting values for the initial stage and 4 optimizations for the final stage. In the example above, the STARTS option specifies that 100 random sets of starting values for the initial stage and 20 final stage optimizations will be used.

In the overall model, the BY statement specifies that the factor f is measured by u1, u2, u3, u4, u5, u6, u7, and u8. The mean of the factor is fixed at zero which implies that the mean is zero in both classes. The factor variance is held equal across classes as the default. The statements in the class-specific parts of the model relax the equality constraints across classes for the factor loadings, factor variance, and the thresholds of the indicators. An explanation of the other commands can be found in Examples 7.1 and 7.3.

EXAMPLE 7.28: TWO-GROUP TWIN MODEL FOR CATEGORICAL OUTCOMES USING MAXIMUM LIKELIHOOD AND PARAMETER CONSTRAINTS

```
TITLE:     this is an example of a two-group twin
           model for categorical outcomes using
           maximum likelihood and parameter
           constraints
DATA:      FILE = ex7.28.dat;
VARIABLE:  NAMES = u1 u2 dz;
           CATEGORICAL = u1 u2;
           CLASSES = cdz (2);
           KNOWNCLASS = cdz (dz = 0 dz = 1);
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
           LINK = PROBIT;
MODEL:     %OVERALL%
           [u1$1-u2$1] (1);
           f1 BY u1;
           f2 BY u2;
           [f1-f2@0];
           f1-f2 (varf);
           %cdz#1%
           f1 WITH f2(covmz);
           %cdz#2%
           f1 WITH f2(covdz);
MODEL CONSTRAINT:
           NEW(a c h);
           varf = a**2 + c**2 + .001;
           covmz = a**2 + c**2;
           covdz = 0.5*a**2 + c**2;
           h = a**2/(a**2 + c**2 + 1);
```



In this example, the model shown in the picture above is estimated. The variables u1 and u2 represent a univariate outcome for each member of a twin pair. Monozygotic and dizygotic twins are considered in a two-group twin model for categorical outcomes using maximum likelihood estimation. Parameter constraints are used to represent the ACE model restrictions. The ACE variance and covariance restrictions are placed on normally-distributed latent response variables, which are also called liabilities, underlying the categorical outcomes. This model is referred to as the threshold model for liabilities (Neale & Cardon, 1992). The monozygotic and dizygotic twin groups are represented by latent classes with known class membership.

The CATEGORICAL option is used to specify which dependent variables are treated as binary or ordered categorical (ordinal) variables in the model and its estimation. In the example above, the variables u1 and u2 are binary or ordered categorical variables. The program determines the number of categories for each indicator. The KNOWNCLASS option identifies cdz as the categorical latent variable for which latent class membership is known. The information in parentheses following the categorical latent variable name defines the known classes using an observed variable. In this example, the observed variable dz is used to define the known classes. The first class consists of the monozygotic twins who have the value 0 on the variable dz. The second class consists of the dizygotic twins who have the value 1 on the variable dz.

By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration

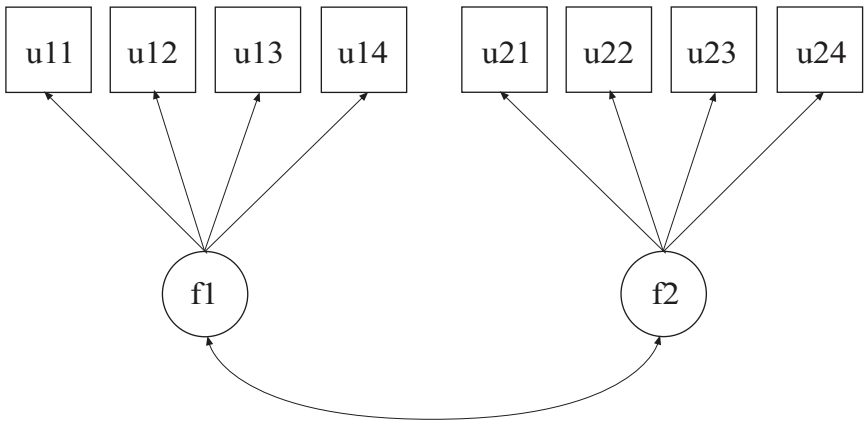
algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, two dimensions of integration are used with 225 integration points. The ESTIMATOR option can be used to select a different estimator. The LINK option is used with maximum likelihood estimation to select a logit or a probit link for models with categorical outcomes. The default is a logit link. In this example, the probit link is used because the threshold model for liabilities uses normally-distributed latent response variables.

In the overall model, the (1) following the first bracket statement specifies that the thresholds of u1 and u2 are held equal across twins. The two BY statements define a factor behind each outcome. This is done because covariances of categorical outcomes are not part of the model when maximum likelihood estimation is used. The covariances of the factors become the covariances of the categorical outcomes or more precisely the covariances of the latent response variables underlying the categorical outcomes. The means of the factors are fixed at zero and their variances are held equal across twins. The variance of each underlying response variable is obtained as the sum of the factor variance plus one where one is the residual variance in the probit regression of the categorical outcome on the factor.

In the MODEL command, labels are defined for three parameters. The label varf is assigned to the variances of f1 and f2. Because they are given the same label, these parameters are held equal. The label covmz is assigned to the covariance between f1 and f2 for the monozygotic twins and the label covdz is assigned to the covariance between f1 and f2 for the dizygotic twins. In the MODEL CONSTRAINT command, the NEW option is used to assign labels to three parameters that are not in the analysis model: a, c, and h. The two parameters a and c are used to decompose the covariances of u1 and u2 into genetic and environmental components. The value .001 is added to the variance of the factors to avoid a singular factor covariance matrix which comes about because the factor variances and covariances are the same. The parameter h does not impose restrictions on the model parameters but is used to compute the heritability estimate and its standard error. This heritability estimate uses the residual variances for the latent response variables which are fixed at one. An explanation of the other commands can be found in Example 7.1.

EXAMPLE 7.29: TWO-GROUP IRT TWIN MODEL FOR FACTORS WITH CATEGORICAL FACTOR INDICATORS USING PARAMETER CONSTRAINTS

```
TITLE:     this is an example of a two-group IRT twin
           model for factors with categorical factor
           indicators using parameter constraints
DATA:      FILE = ex7.29.dat;
VARIABLE:  NAMES = u11-u14 u21-u24 dz;
           CATEGORICAL = u11-u24;
           CLASSES = cdz (2);
           KNOWNCLASS = cdz (dz = 0 dz = 1);
ANALYSIS:  TYPE = MIXTURE;
           ALGORITHM = INTEGRATION;
MODEL:     %OVERALL%
           f1 BY u11
                 u12-u14 (lam2-lam4);
           f2 BY u21
                 u22-u24 (lam2-lam4);
           [f1-f2@0];
           f1-f2 (var);
           [u11$1-u14$1] (t1-t4);
           [u21$1-u24$1] (t1-t4);
           %cdz#1%
           f1 WITH f2(covmz);
           %cdz#2%
           f1 WITH f2(covdz);
MODEL CONSTRAINT:
           NEW(a c e h);
           var = a**2 + c**2 + e**2;
           covmz = a**2 + c**2;
           covdz = 0.5*a**2 + c**2;
           h = a**2/(a**2 + c**2 + e**2);
```



In this example, the model shown in the picture above is estimated. The factors f1 and f2 represent a univariate variable for each member of the twin pair. Monozygotic and dizygotic twins are considered in a two-group twin model for factors with categorical factor indicators using parameter constraints and maximum likelihood estimation. Parameter constraints are used to represent the ACE model restrictions. The ACE variance and covariance restrictions are placed on two factors instead of two observed variables as in Example 7.28. The relationships between the categorical factor indicators and the factors are logistic regressions. Therefore, the factor model for each twin is a two-parameter logistic Item Response Theory model (Muthén, Asparouhov, & Rebollo, 2006). The monozygotic and dizygotic twin groups are represented by latent classes with known class membership.

By specifying ALGORITHM=INTEGRATION, a maximum likelihood estimator with robust standard errors using a numerical integration algorithm will be used. Note that numerical integration becomes increasingly more computationally demanding as the number of factors and the sample size increase. In this example, two dimensions of integration are used with 225 integration points. The ESTIMATOR option can be used to select a different estimator.

In the overall model, the two BY statements specify that f1 is measured by u11, u12, u13, and u14 and that f2 is measured by u21, u22, u23, and u24. The means of the factors are fixed at zero. In the class-specific models, the threshold of the dz variable is fixed at 15 in class one and -15 in class 2.

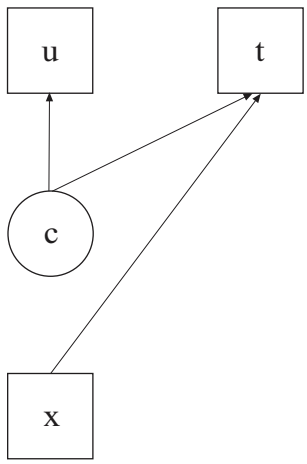
In the MODEL command, labels are defined for nine parameters. The list function can be used when assigning labels. The label lam2 is assigned to the factor loadings for u12 and u22; the label lam3 is assigned to the factor loadings for u13 and u23; and the label lam4 is assigned to the factor loadings for u14 and u24. Factor loadings with the same label are held equal. The label t1 is assigned to the thresholds of u11 and u21; the label t2 is assigned to the thresholds of u12 and u22; the label t3 is assigned to the thresholds of u13 and u23; and the label t4 is assigned to the thresholds of u14 and u24. Parameters with the same label are held equal. The label covmz is assigned to the covariance between f1 and f2 for the monozygotic twins and the label covdz is assigned to the covariance between f1 and f2 for the dizygotic twins.

In the MODEL CONSTRAINT command, the NEW option is used to assign labels to four parameters that are not in the analysis model: a, c, e, and h. The three parameters a, c, and e are used to decompose the variances and covariances of f1 and f2 into genetic and environmental components. The parameter h does not impose restrictions on the model parameters but is used to compute the heritability estimate and its standard error. An explanation of the other commands can be found in Examples 7.1 and 7.28.

EXAMPLE 7.30: CONTINUOUS-TIME SURVIVAL ANALYSIS USING A COX REGRESSION MODEL TO ESTIMATE A TREATMENT EFFECT

```
TITLE:     this is an example of continuous-time
           survival analysis using a Cox regression
           model to estimate a treatment effect
DATA:      FILE = ex7.30.dat;
VARIABLE:  NAMES are t u x tcent class;
           USEVARIABLES = t-tcent;
           SURVIVAL = t;
           TIMECENSORED = tcent;
           CATEGORICAL = u;
           CLASSES = c (2);
ANALYSIS:  TYPE = MIXTURE;
```

```
MODEL:
            %OVERALL%
            t ON x;
            %c#1%
            [u$1@15];
            [t@0];
            %c#2%
            [u$1@-15];
            [t];
OUTPUT:     TECH1 LOGRANK;
PLOT:       TYPE = PLOT2;
```



In this example, the continuous-time survival analysis model shown in the picture above is estimated. The model is similar to Larsen (2004). A treatment and a control group are analyzed as two known latent classes. The baseline hazards are held equal across the classes and the treatment effect is expressed as the intercept of the survival variable in the treatment group. For applications of this model, see Muthén et al. (2009).

The CATEGORICAL option is used to specify that the variable u is a binary variable. This variable is a treatment dummy variable where zero represents the control group and one represents the treatment group. In this example, the categorical latent variable c has two classes. In the MODEL command, in the model for class 1, the threshold for u is fixed at 15 so that the probability that u equals one is zero. By this

specification, class 1 is the control group. In the model for class 2, the threshold for u is fixed at -15 so that the probability that u equals one is one. By this specification, class 2 is the treatment group. In the overall model, the ON statement describes the Cox regression for the survival variable t on the covariate x. In class 1, the intercept in the Cox regression is fixed at zero. In class 2, it is free. This intercept represents the treatment effect. The LOGRANK option of the OUTPUT command provides a logrank test of the equality of the treatment and control survival curves (Mantel, 1966). By specifying PLOT2 in the PLOT command, the following plots are obtained:

- Kaplan-Meier curve
- Sample log cumulative hazard curve
- Estimated baseline hazard curve
- Estimated baseline survival curve
- Estimated log cumulative baseline curve
- Kaplan-Meier curve with estimated baseline survival curve
- Sample log cumulative hazard curve with estimated log cumulative baseline curve

An explanation of the other commands can be found in Example 7.1.