Latent Variable Modeling Using Mplus: Day 2

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Mplus
www.statmodel.com

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Mplus Integrates A Multitude Of Analysis Types Using The Unifying Theme Of Latent Variables

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Growth modeling
- Latent class analysis
- Latent transition analysis (Hidden Markov modeling)
- Growth mixture modeling
- Survival analysis
- Missing data modeling
- Multilevel analysis
- Complex survey data analysis
- Bayesian analysis
- Causal inference
Mplus Integrates A Multitude Of Analysis Types Using The Unifying Theme Of Latent Variables

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- Structural equation modeling
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- Survival analysis
- Latent class analysis
1. Overview Of Analysis With Categorical Latent Variables

Used to capture heterogeneity when individuals come from different unobserved subpopulations

Application Areas

- Cross-sectional data
  - Medical and psychiatric diagnosis such as Alzheimer’s disease, schizophrenia, depression, alcoholism
  - Market segmentation
  - Mastery in educational development

- Longitudinal data
  - Multiple disease processes such as prostate-specific antigen development
  - Developmental pathways such as adolescent-limited versus life-course persistent antisocial behavior
Analysis Methods

- Regression mixture models - Modeling of counts, randomized interventions with non-compliance
- Latent class analysis with and without covariates
- Latent transition analysis
- Latent class growth analysis
- Growth mixture modeling
- Survival mixture modeling

- Alzheimer’s Disease (AD) Neuroimaging Initiative
- Adults aged 55 to 90 years
- 3 groups based on cognitive tests and clinical ratings:
  - Mild AD
  - Mild cognitive impairment
  - Cognitively normal
- Measure of cerebrospinal fluid-derived $\beta$-amyloid protein
Cerebrospinal Fluid Protein Distributions: A 2-Class Mixture

**Figure 1.** Mixture model classification for cerebrospinal fluid–derived β-amyloid protein 1-42 (CSF Aβ1-42). Results are presented as a histogram of observed counts overlaid with the 2 mixture distributions and the joined distribution based on the mixture proportion.

**Figure 2.** Cerebrospinal fluid–derived β-amyloid protein 1-42 (CSF Aβ1-42) mixture model applied to the clinically diagnosed subject groups. AD indicates Alzheimer disease; MCI, mild cognitive impairment.
Mixture Modeling Prototypes, Continued
2. Regression With A Count Dependent Variable

- Poisson
- Zero-inflated Poisson (ZIP)
- Negative binomial
- Mixture ZIP
- ZI negative binomial
- Mixture negative binomial
2.1 Poisson Regression

A Poisson distribution for a count variable $u_i$ has

$$P(u_i = r) = \frac{\lambda_i^r e^{-\lambda_i}}{r!},$$

where $u_i = 0, 1, 2, \ldots$

$\lambda$ is the rate at which a rare event occurs (rate = mean count)

Regression equation for the log rate:

$$e \log \lambda_i = \ln \lambda_i = \beta_0 + \beta_1 x_i$$
A Poisson variable has variance = mean = $\lambda$, but count data often have variance $> \text{mean}$ due to preponderance of zeros. Zero-inflated Poisson modeling avoids this restriction.

Alcohol abuse example: How many times in the last month did you drink 5 or more drinks at one occasion?

- Two classes of subjects: Drinkers and Non-drinkers
- A zero observation may be obtained because the subject is a non-drinker or because he/she is a drinker but did not drink 5 or more drinks at one occasion during the last month
- ”Mixture at zero”

ZIP is a model with two latent classes:
- $\pi = P$ (being in the zero class where only $u = 0$ is seen)
- $1 - \pi = P$ (not being in the zero class with $u$ following a Poisson distribution)
A mixture at zero ($\pi$ is the probability of being in the zero class):

- $P(u = 0) = \pi + (1 - \pi)e^{-\lambda}$, where $e^{-\lambda} = \text{Poisson for 0 count}$
- ZIP mean count: $\lambda (1 - \pi)$
- ZIP variance: $\lambda (1 - \pi)(1 + \lambda \times \pi)$
- The ZIP model implies two regressions:

$$\text{logit}(\pi_i) = \gamma_0 + \gamma_1 x_i,$$
$$\ln \lambda_i = \beta_0 + \beta_1 x_i$$
Unobserved heterogeneity $e_i$ is added to the Poisson model

$$\ln \lambda_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$
where $\exp(\varepsilon) \sim \Gamma$

Poisson assumes

$E(u_i|x_i) = \lambda_i$

$V(u_i|x_i) = \lambda_i$

Negative binomial assumes

$E(u_i|x_i) = \lambda_i$

$V(u_i|x_i) = \lambda_i (1 + \lambda_i \alpha)$

NB with $\alpha = 0$ gives Poisson. When the dispersion parameter $\alpha > 0$, the NB model gives substantially higher probability for low counts and somewhat higher probability for high counts than Poisson.

Further variations are zero-inflated NB and zero-truncated NB (hurdle model or two-part model).
Allowing any number of latent classes, not only a mixture at zero:

\[ \text{logit} (\pi_i) = \gamma_0 + \gamma_1 x_i, \]

\[ \ln \lambda_{i|C=c_i} = \beta_{0c} + \beta_{1c} x_i. \]

An equivalent generalization of zero-inflated negative binomial is possible.
2.4 Counts Of Marital Affairs

Dependent variable: Number of affairs reported in the last year
Covariates: Having kids, marital happiness, religiosity, years married
## Model Alternatives For Counts Of Marital Affairs ($n = 601$)

<table>
<thead>
<tr>
<th>Model</th>
<th>Log-Likelihood</th>
<th># of Parameters</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>-1,399.913</td>
<td>13</td>
<td>2883</td>
</tr>
<tr>
<td>Negative Binomial</td>
<td>-724.240</td>
<td>14</td>
<td>1538</td>
</tr>
<tr>
<td>Zero-inflated Poisson</td>
<td>-783.002</td>
<td>14</td>
<td>1656</td>
</tr>
<tr>
<td>Zero-inflated negative binomial</td>
<td>-718.064</td>
<td>15</td>
<td>1532</td>
</tr>
<tr>
<td>2-class Poisson mixture</td>
<td>-728.001</td>
<td>15</td>
<td>1552</td>
</tr>
<tr>
<td>2-class negative binomial mixture</td>
<td>-718.064</td>
<td>16</td>
<td>1539</td>
</tr>
<tr>
<td>2-class zero-inflated Poisson</td>
<td><strong>-700.718</strong></td>
<td>16</td>
<td>1504</td>
</tr>
<tr>
<td>Model</td>
<td>Log-Likelihood</td>
<td># of Parameters</td>
<td>BIC</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>----------------</td>
<td>-----------------</td>
<td>-------</td>
</tr>
<tr>
<td>2-class zero-inflated negative binomial</td>
<td>-700.718</td>
<td>17</td>
<td>1510</td>
</tr>
<tr>
<td>2-class negative binomial hurdle</td>
<td>-726.039</td>
<td>15</td>
<td>1548</td>
</tr>
<tr>
<td>Poisson with normal residual</td>
<td>-735.953</td>
<td>14</td>
<td>1561</td>
</tr>
</tbody>
</table>
TITLE: Hilbe page 112 example
DATA: FILE = affairs1.dat;
VARIABLE: NAMES = ID
         male age yrsmarr kids relig educ occup ratemarr naffairs affair
         vryhap hapavg avgmarr unhap vryrel smerel slghtrel notrel;
USEVAR = naffairs kids vryhap hapavg avgmarr vryrel smerel slghtrel notrel yrsmarr3 yrsmarr4 yrsmarr5 yrsmarr6;
COUNT = naffairs(pi);
CLASSES = c(2);
DEFINE: IF (yrsmarr==4) THEN yrsmarr3=1 ELSE yrsmarr3=0;
         IF (yrsmarr==7) THEN yrsmarr4=1 ELSE yrsmarr4=0;
         IF (yrsmarr==10) THEN yrsmarr5=1 ELSE yrsmarr5=0;
         IF (yrsmarr==15) THEN yrsmarr6=1 ELSE yrsmarr6=0;
ANALYSIS: TYPE = MIXTURE;
         ESTIMATOR = ML;
         PROCESSORS = 8;
MODEL:

%OVERALL%
naffairs ON kids-yrsmarr6 (p1-p12);
! It is also possible to model the logit of the probability
! of being in the zero class:
! naffairs#1 ON kids-yrsmarr6;
! Also possible: c ON kids-yrsmarr6;

MODEL CONSTRAINT:

NEW(e1-e12);
DO(1,12) e# = exp(p#);

OUTPUT: TECH1;
3. Estimating Treatment Effects in Randomized Trials with Non-Compliance


Potential outcomes, principal stratification, latent classes (mixtures)
Randomized Trials With Non-Compliance

- Tx group (compliance status observed)
  - Compliers
  - Noncompliers
- Control group (compliance status unobserved)
  - Compliers
  - Noncompliers

Compliers and Noncompliers are typically not randomly equivalent subgroups.

Four approaches to estimating treatment effects:

1. Tx versus Control (Intent-To-Treat; ITT)
2. Tx Compliers versus Control (Per Protocol)
3. Tx Compliers versus Tx NonCompliers + Control (As-Treated)
4. Mixture analysis (Complier Average Causal Effect; CACE):
   - Tx Compliers versus Control Compliers
   - Tx NonCompliers versus Control NonCompliers
- $z$ is a 0/1 dummy variable indicating treatment assignment
- $c$ is a latent class variable (Complier and Non-Complier)
- $u$ is a categorical variable with categories Show and No-Show.
  - $u$ is missing for the control group
  - $u$ is identical to $c$ for the treatment group ($c$ observed for Tx)
The JOBS data are from a Michigan University Prevention Research Center study of interventions aimed at preventing poor mental health of unemployed workers and promoting high quality of reemployment. The intervention consisted of five half-day training seminars that focused on problem solving, decision making group processes, and learning and practicing job search skills. The control group received a booklet briefly describing job search methods and tips. Respondents were recruited from the Michigan Employment Security Commission. After a series of screening procedures, 1801 were randomly assigned to treatment and control conditions. Of the 1249 in the treatment group, only 54% participated in the treatment.

The variables collected in the study include depression scores and outcome measures related to reemployment. Background variables include demographic and psychosocial variables.
Data for the analysis include the outcome variable of depression and the background variables of treatment status, age, education, marital status, SES, ethnicity, a risk score for depression, a pre-intervention depression score, a measure of motivation to participate, and a measure of assertiveness. A subset of 502 individuals classified as having high-risk of depression were analyzed.

The analysis replicates that of Little & Yau (1998).
TITLE: Complier Average Causal Effect (CACE) estimation in a randomized trial.
Data from the JOBS II intervention trial, courtesy of Richard Price and Amiram Vinokur, University of Michigan.
The analysis below replicates that of:
Treatment of no-shows using Rubin’s causal model.
Psychological Methods, 3, 147-159.

DATA: FILE = jobs with u.dat;
VARIABLE: NAMES = depress risk Tx depbase age motivate educ assert single econ nonwhite u;
CATEGORICAL = u; ! u=1 show, u=0 no-show
MISSING = ALL(999);
CLASSES = c(2);

ANALYSIS: TYPE = MIXTURE;
STARTS = 100 20;
PROCESSORS = 8;
MODEL:  

%OVERALL%
depress ON Tx risk depbase;
c ON age educ motivate econ assert single nonwhite;
%c#1%
! c#1 is the complier class (shows)
[u$1@-15]; ! P(u = 1) = 1
! [depress]; different across class as the default
%c#2%
! c#2 is the noncomplier class (no-shows)
[u$1@15]; ! P(u = 1) = 0
! [depress]; different across class as the default
depress ON Tx@0;

OUTPUT:  TECH1 TECH8;
## Final Class Counts And Proportions For the Latent Classes Based On The Estimated Model

<table>
<thead>
<tr>
<th>Latent Classes</th>
<th>271.93480</th>
<th>0.54170</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>230.06520</td>
<td>0.45830</td>
</tr>
</tbody>
</table>

## Classification Quality

| Entropy | 0.727 |

## Average Latent Class Probabilities For Most Likely Latent Class Membership (Row) By Latent Class (Column)

<table>
<thead>
<tr>
<th>Latent Classes</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.900</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>0.097</td>
<td>0.903</td>
</tr>
<tr>
<td>Latent Class 1</td>
<td>Estimates</td>
<td>S.E.</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------</td>
<td>------</td>
</tr>
<tr>
<td>depress ON</td>
<td>-0.310</td>
<td>0.130</td>
</tr>
<tr>
<td>tx</td>
<td>0.912</td>
<td>0.247</td>
</tr>
<tr>
<td>risk</td>
<td>-1.463</td>
<td>0.181</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.812</td>
<td>0.299</td>
</tr>
<tr>
<td>Thresholds</td>
<td>-15.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Residual Variances</td>
<td>0.506</td>
<td>0.037</td>
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</tbody>
</table>
### Output For JOBS Example, Continued

<table>
<thead>
<tr>
<th>Latent Class 2</th>
<th>Estimates</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
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<tbody>
<tr>
<td>depress ON</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tx</td>
<td>0.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>risk</td>
<td>0.912</td>
<td>0.247</td>
<td>3.685</td>
<td>0.000</td>
</tr>
<tr>
<td>depbase</td>
<td>-1.463</td>
<td>0.181</td>
<td>-8.077</td>
<td>0.000</td>
</tr>
<tr>
<td>Intercepts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depress</td>
<td>1.633</td>
<td>0.273</td>
<td>5.977</td>
<td>0.000</td>
</tr>
<tr>
<td>Thresholds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u$1</td>
<td>15.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>Residual Variances</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depress</td>
<td>0.506</td>
<td>0.037</td>
<td>13.742</td>
<td>0.000</td>
</tr>
<tr>
<td>Categorical Latent Variables</td>
<td>Estimates</td>
<td>S.E.</td>
<td>Est./S.E.</td>
<td>Two-Tailed P-Value</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------</td>
<td>------</td>
<td>-----------</td>
<td>-------------------</td>
</tr>
<tr>
<td>c#1 ON</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>age</td>
<td>0.079</td>
<td>0.015</td>
<td>5.184</td>
<td>0.000</td>
</tr>
<tr>
<td>educ</td>
<td>0.300</td>
<td>0.068</td>
<td>4.390</td>
<td>0.000</td>
</tr>
<tr>
<td>motivate</td>
<td>0.667</td>
<td>0.157</td>
<td>4.243</td>
<td>0.000</td>
</tr>
<tr>
<td>econ</td>
<td>-0.159</td>
<td>0.152</td>
<td>-1.045</td>
<td>0.296</td>
</tr>
<tr>
<td>assert</td>
<td>-0.376</td>
<td>0.143</td>
<td>-2.631</td>
<td>0.009</td>
</tr>
<tr>
<td>single</td>
<td>0.540</td>
<td>0.283</td>
<td>1.908</td>
<td>0.056</td>
</tr>
<tr>
<td>nonwhite</td>
<td>-0.499</td>
<td>0.317</td>
<td>-1.571</td>
<td>0.116</td>
</tr>
<tr>
<td>Intercepts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c#1</td>
<td>-8.740</td>
<td>1.590</td>
<td>-5.498</td>
<td>0.000</td>
</tr>
</tbody>
</table>
4. Latent Class Analysis

- **Class 1**
- **Class 2**
- **Class 3**
- **Class 4**

Item Probability

- **inatt1**
- **inatt2**
- **hyper1**
- **hyper2**
Latent Class Analysis
Alcohol Dependence Criteria, NLSY 1989 (n = 8313)

<table>
<thead>
<tr>
<th>Prevalence</th>
<th>Latent Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Two-class solution¹</td>
</tr>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>I</td>
<td>0.78</td>
</tr>
<tr>
<td>DSM-III-R-Criterion</td>
<td>Conditional Probability of Fulfilling a Criterion</td>
</tr>
<tr>
<td>Withdrawal</td>
<td>0.00</td>
</tr>
<tr>
<td>Tolerance</td>
<td>0.01</td>
</tr>
<tr>
<td>Larger</td>
<td>0.15</td>
</tr>
<tr>
<td>Cut down</td>
<td>0.00</td>
</tr>
<tr>
<td>Time spent</td>
<td>0.00</td>
</tr>
<tr>
<td>Major role-Hazard</td>
<td>0.03</td>
</tr>
<tr>
<td>Give up</td>
<td>0.00</td>
</tr>
<tr>
<td>Relief</td>
<td>0.00</td>
</tr>
<tr>
<td>Continue</td>
<td>0.00</td>
</tr>
</tbody>
</table>

¹Likelihood ratio chi-square fit = 1779 with 492 degrees of freedom
²Likelihood ratio chi-square fit = 448 with 482 degrees of freedom
### Latent Class Membership By Number Of DSM-III-R Alcohol Dependence Criteria Met (n=8313)

<table>
<thead>
<tr>
<th>Number of Criteria Met</th>
<th>%</th>
<th>Two-class solution</th>
<th>Three-class solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>0</td>
<td>64.2</td>
<td>5335</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>14.0</td>
<td>1161</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
<td>0</td>
<td>845</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>0</td>
<td>469</td>
</tr>
<tr>
<td>4</td>
<td>2.6</td>
<td>0</td>
<td>213</td>
</tr>
<tr>
<td>5</td>
<td>1.4</td>
<td>0</td>
<td>116</td>
</tr>
<tr>
<td>6</td>
<td>0.8</td>
<td>0</td>
<td>68</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0</td>
<td>39</td>
</tr>
<tr>
<td>9</td>
<td>0.3</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>%</td>
<td>100.0</td>
<td>78.1</td>
<td>21.9</td>
</tr>
</tbody>
</table>
LCA Item Profiles For NLSY Alcohol Criteria
TITLE: Alcohol LCA M & M (1993)
DATA: FILE = bengt03_spread.dat;
VARIABLE: NAMES = u1-u9;
          CATEGORICAL = u1-u9;
          CLASSES = c(3);
ANALYSIS: TYPE = MIXTURE;
PLOT: TYPE = PLOT3;
       SERIES = u1-u9(*);
Factor mixture analysis
  - Generalized factor analysis
  - Generalized latent class analysis

Latent Class Analysis

a. Item Profiles

b. Model Diagram
a. Item Response Curves

b. Model Diagram

Item Probability

Factor (f)

Inatt1 → inatt2 → hyper1 → hyper2

x → c → hyper1 → hyper2 → inatt1 → inatt2
Latent Class, Factor, And Factor Mixture Analysis
Alcohol Dependence Criteria, NLSY 1989 (n = 8313)

<table>
<thead>
<tr>
<th>DSM-III-R criterion</th>
<th>Two-class solution</th>
<th>Three-class solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Prevalence</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>DSM-III-R criterion</td>
<td>conditional probability of fulfilling a criterion</td>
<td></td>
</tr>
<tr>
<td>Withdrawal</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>Tolerance</td>
<td>0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>Larger</td>
<td>0.15</td>
<td>0.96</td>
</tr>
<tr>
<td>Cut down</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>Time spent</td>
<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>Major role-hazard</td>
<td>0.03</td>
<td>0.83</td>
</tr>
<tr>
<td>Give up</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Relief</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td>Continue</td>
<td>0.00</td>
<td>0.24</td>
</tr>
</tbody>
</table>

1 Likelihood ratio chi-square fit = 1779 with 492 degrees of freedom
2 Likelihood ratio chi-square fit = 448 with 482 degrees of freedom
LCA, 3 classes:  \( \log L = -14,139 \), 29 parameters, BIC = 28,539

FA, 2 factors:  \( \log L = -14,083 \), 26 parameters, BIC = 28,401

FMA 2 classes, 1 factor, loadings invariant:
\( \log L = -14,054 \), 29 parameters, BIC = 28,370

Models can be compared with respect to fit to the data using TECH10:

- Standardized bivariate residuals
- Standardized residuals for most frequent response patterns
<table>
<thead>
<tr>
<th>Obs. Freq.</th>
<th>LCA 3c</th>
<th>FA 2f</th>
<th>FMA 1f, 2c</th>
</tr>
</thead>
<tbody>
<tr>
<td>5335</td>
<td>5332</td>
<td>-0.07</td>
<td>5307</td>
</tr>
<tr>
<td>941</td>
<td>945</td>
<td>0.12</td>
<td>985</td>
</tr>
<tr>
<td>601</td>
<td>551</td>
<td>-2.22</td>
<td>596</td>
</tr>
<tr>
<td>217</td>
<td>284</td>
<td>4.04</td>
<td>211</td>
</tr>
<tr>
<td>155</td>
<td>111</td>
<td>-4.16</td>
<td>118</td>
</tr>
<tr>
<td>149</td>
<td>151</td>
<td>0.15</td>
<td>168</td>
</tr>
<tr>
<td>65</td>
<td>68</td>
<td>0.41</td>
<td>46</td>
</tr>
<tr>
<td>49</td>
<td>52</td>
<td>0.42</td>
<td>84</td>
</tr>
<tr>
<td>48</td>
<td>54</td>
<td>0.81</td>
<td>44</td>
</tr>
<tr>
<td>47</td>
<td>40</td>
<td>-1.09</td>
<td>45</td>
</tr>
</tbody>
</table>

**Bolded** entries are significant at the 5% level.
DATA: FILE = bengt05_spread.dat;
VARIABLE: NAMES = u1-u9;
          CATEGORICAL = u1-u9;
          CLASSES = c(2);
ANALYSIS: TYPE = MIXTURE;
          ALGORITHM = INTEGRATION;
          STARTS = 200 10; STITER = 20;
          ADAPTIVE = OFF;
          PROCESSORS = 8;
MODEL: %OVERALL%
   f BY u1-u9;
   f*1; [f@0];
   %c#1%
   [u1$1-u9$1];
   f*1;
   %c#2%
   [u1$1-u9$1];
   f*1;

OUTPUT: TECH1 TECH8 TECH10;

PLOT: TYPE = PLOT3;
 SERIES = u1-u9(*);
UCLA clinical sample of 425 males ages 5-18, all with ADHD diagnosis

Subjects assessed by clinicians:
1) direct interview with child (> 7 years),
2) interview with mother about child

KSADS: Nine inattentiveness items, nine hyperactivity items; dichotomously scored

Families with at least 1 ADHD affected child

Parent data, candidate gene data on sib pairs

What types of ADHD does a treatment population show?
<table>
<thead>
<tr>
<th>Inattentiveness items:</th>
<th>Hyperactivity items:</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Difficulty sustaining attn on tasks/play’</td>
<td>'Difficulty remaining seated’</td>
</tr>
<tr>
<td>'Easily distracted’</td>
<td>'Fidgets’</td>
</tr>
<tr>
<td>'Makes a lot of careless mistakes’</td>
<td>'Runs or climbs excessively’</td>
</tr>
<tr>
<td>'Doesn’t listen’</td>
<td>'Difficulty playing quietly’</td>
</tr>
<tr>
<td>'Difficulty following instructions’</td>
<td>'Blurts out answers’</td>
</tr>
<tr>
<td>'Difficulty organizing tasks’</td>
<td>'Difficulty waiting turn’</td>
</tr>
<tr>
<td>'Dislikes/avoids tasks’</td>
<td>'Interrupts or intrudes’</td>
</tr>
<tr>
<td>'Loses things’</td>
<td>'Talks excessively’</td>
</tr>
<tr>
<td>'Forgetful in daily activities’</td>
<td>'Driven by motor’</td>
</tr>
<tr>
<td>Model</td>
<td>Likelihood</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>LCA - 2c</td>
<td>-3650</td>
</tr>
<tr>
<td>LCA - 3c</td>
<td><strong>-3545</strong></td>
</tr>
<tr>
<td>LCA - 4c</td>
<td>-3499</td>
</tr>
<tr>
<td>LCA - 5c</td>
<td>-3464</td>
</tr>
<tr>
<td>LCA - 6c</td>
<td><strong>-3431</strong></td>
</tr>
<tr>
<td>LCA - 7c</td>
<td>-3413</td>
</tr>
</tbody>
</table>

LCA-3c is best by BIC and LCA-6c is best by BLRT
Three-Class And Six-Class LCA Item Profiles

LCA- 3c

LCA -6c
<table>
<thead>
<tr>
<th>Model</th>
<th>Likelihood</th>
<th># Parameters</th>
<th>BIC</th>
<th>BLRT p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCA - 2c</td>
<td>-3650</td>
<td>37</td>
<td>7523</td>
<td>0.00</td>
</tr>
<tr>
<td>LCA - 3c</td>
<td>-3545</td>
<td>56</td>
<td>7430</td>
<td>0.00</td>
</tr>
<tr>
<td>LCA - 4c</td>
<td>-3499</td>
<td>75</td>
<td>7452</td>
<td>0.00</td>
</tr>
<tr>
<td>LCA - 5c</td>
<td>-3464</td>
<td>94</td>
<td>7496</td>
<td>0.00</td>
</tr>
<tr>
<td>LCA - 6c</td>
<td>-3431</td>
<td>113</td>
<td>7547</td>
<td>0.00</td>
</tr>
<tr>
<td>LCA - 7c</td>
<td>-3413</td>
<td>132</td>
<td>7625</td>
<td>0.27</td>
</tr>
<tr>
<td>EFA - 2f</td>
<td>-3505</td>
<td>53</td>
<td>7331</td>
<td></td>
</tr>
</tbody>
</table>

The EFA model is better than LCA-3c, but no classification of individuals is obtained.
### The Latent Structure Of ADHD: Model Results

| Model               | Likelihood | # Parameters | BIC  | BLRT p value
|---------------------|------------|--------------|------|----------------
| LCA - 2c            | -3650      | 37           | 7523 | 0.00           |
| LCA - 3c            | **-3545**  | 56           | 7430 | **0.00**       |
| LCA - 4c            | -3499      | 75           | 7452 | 0.00           |
| LCA - 5c            | -3464      | 94           | 7496 | 0.00           |
| LCA - 6c            | **-3431**  | 113          | 7547 | **0.00**       |
| LCA - 7c            | -3413      | 132          | 7625 | 0.27           |
| EFA - 2f            | -3505      | 53           | 7331 |                |
| FMA - 2c, 2f        | -3461      | 59           | 7280 |                |
| FMA - 2c, 2f        |            |              |      |                |
| Class-varying       | -3432      | 75           | 7318 | $\chi^2$-diff (16)=58   |
| Factor loadings     |            |              |      | p < 0.01       |
Item Profiles For Three-Class LCA, Six-Class LCA And Two-Class, Two-Factor FMA

LCA- 3c

LCA- 6c

FMA- 2c, 2f
The Antisocial Behavior (ASB) data were taken from the National Longitudinal Survey of Youth (NLSY) that is sponsored by the Bureau of Labor Statistics. These data are made available to the public by Ohio State University. The data were obtained as a multistage probability sample with oversampling of blacks, Hispanics, and economically disadvantaged non-blacks and non-Hispanics.

Data for the analysis include 17 antisocial behavior items that were collected in 1980 when respondents were between the ages of 16 and 23 and the background variables of age, gender, and ethnicity. The ASB items assessed the frequency of various behaviors during the past year. A sample of 7,326 respondents has complete data on the antisocial behavior items and the background variables of age, gender, and ethnicity.
Following is a list of the 17 items:

<table>
<thead>
<tr>
<th>Property offense:</th>
<th>Person offense:</th>
<th>Drug offense:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damaged property</td>
<td>Fighting</td>
<td>Use marijuana</td>
</tr>
<tr>
<td>Shoplifting</td>
<td>Use of force</td>
<td>Use other drugs</td>
</tr>
<tr>
<td>Stole &lt; $50</td>
<td>Seriously threaten</td>
<td>Sold marijuana</td>
</tr>
<tr>
<td>Stole &gt; $50</td>
<td>Intent to injure</td>
<td>Sold hard drugs</td>
</tr>
<tr>
<td>”Con” someone</td>
<td>Gambling operation</td>
<td></td>
</tr>
<tr>
<td>Take auto</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Broken into building</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Held stolen goods</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The items were dichotomized 0/1 with 0 representing never in the last year.

Are there different groups of people with different ASB profiles?
TITLE: LCA of 9 ASB items with three covariates
DATA: FILE = asb.dat;
        FORMAT = 34x 51f2;
VARIABLE: NAMES = property fight shoplift lt50 gt50 force threat injure pot drug soldpot solddrug con auto bldg goods gambling dsm1-dsm22 male black hisp single divorce dropout college onset f1 f2 f3 age94;
        USEVARIABLES = property fight shoplift lt50 threat pot drug con goods age94 male black;
        CLASSES = c(4);
        CATEGORICAL = property-goods;
ANALYSIS: TYPE = MIXTURE;
MODEL: %OVERALL%
c#1-c#3 ON age94 male black;
%c#1% !Not needed – High class
[property$1-goods$1*0]; !Not needed
%c#2% !Not needed – Drug class (pot, drugs)
[property$1-goods$1*1]; !Not needed
%c#3% !Not needed – Person class (fight, threaten)
[property$1-goods$1*2]; !Not needed
%c#4% !Not needed – Low class
[property$1-goods$1*3]; !Not needed
OUTPUT: TECH1 TECH8;
### Output: Antisocial Behavior (ASB) Items With Covariates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c#1) ON</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>age94</td>
<td>-0.285</td>
<td>0.028</td>
<td>-10.045</td>
</tr>
<tr>
<td>male</td>
<td>2.578</td>
<td>0.151</td>
<td>17.086</td>
</tr>
<tr>
<td>black</td>
<td>0.158</td>
<td>0.139</td>
<td>1.141</td>
</tr>
<tr>
<td>(c#2) ON</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>age94</td>
<td>0.069</td>
<td>0.022</td>
<td>3.182</td>
</tr>
<tr>
<td>male</td>
<td>0.187</td>
<td>0.110</td>
<td>1.702</td>
</tr>
<tr>
<td>black</td>
<td>-0.606</td>
<td>0.139</td>
<td>-4.357</td>
</tr>
<tr>
<td>(c#3) ON</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>age94</td>
<td>-0.317</td>
<td>0.028</td>
<td>-11.311</td>
</tr>
<tr>
<td>male</td>
<td>1.459</td>
<td>0.101</td>
<td>14.431</td>
</tr>
<tr>
<td>black</td>
<td>0.999</td>
<td>0.117</td>
<td>8.513</td>
</tr>
</tbody>
</table>
1-step analysis versus 3-step analysis (analyze-classify-analyze)
However, the one-step approach has certain disadvantages. The first is that it may sometimes be impractical, especially when the number of potential covariates is large, as will typically be the case in a more exploratory study. Each time that a covariate is added or removed not only the prediction model but also the measurement model needs to be reestimated. A second disadvantage is that it introduces additional model building problems, such as whether one should decide about the number of classes in a model with or without covariates. Third, the simultaneous approach does not fit with the logic of most applied researchers, who view introducing covariates as a step that comes after the classification model has been built. Fourth, it assumes that the classification model is built in the same stage of a study as the model used to predict the class membership, which is not necessarily the case.
Substantive question: Should the latent classes be defined by the indicators alone or also by covariates and distals?

Example: Study of genotypes influencing phenotypes.

Phenotypes may be observed indicators of mental illness such as DSM criteria. The interest is in finding latent classes of subjects and then trying to see if certain genotype variables influence class membership.

Possible objection to 1-step: If the genotypes are part of deciding the latent classes, the assessment of the strength of relationship is compromised.

3-step: Determine the latent classes based on only phenotype information. Then classify subjects. Then relate the classification to the genotypes.
Step 1: Do LCA on the latent class indicators
Step 2: Classify subjects into most likely class
Step 3: Regress the nominal most likely class variable on covariates

Problem: The Step 3 regression ignores the misclassification of the nominal observed variable being different from the latent class variable. This causes biased Step 3 estimates and SEs. The biases increase when entropy (classification quality: 0-1) decreases.
Prior to Mplus Version 7: Pseudo-class (PC) approach. Estimate LCA model, impute $C$, regress imputed $C$ on $X$

New improved method in Mplus Version 7: 3-step approach

1. Estimate the LCA model
2. Create a nominal most likely class variable $N$
3. Use a mixture model for $N$, $C$ and $X$, where $N$ is a $C$ indicator with measurement error rates prefixed at the uncertainty rate of $N$ estimated in the step 1 LCA analysis


Vermunt (2010). Latent Class Modeling with Covariates: Two Improved Three-Step Approaches. Political Analysis, 18, 450-469
VARIABLE:  NAMES = u1-u5 x;
           CATEGORICAL = u1-u5;
           CLASSES = c(2);
           AUXILIARY = x(R3STEP);
DATA:     FILE = 1.dat;
ANALYSIS: TYPE = MIXTURE;
MODEL:    !no model is needed, LCA is default
### TESTS OF CATEGORICAL LATENT VARIABLE MULTINOMIAL LOGISTIC REGRESSIONS

#### THE 3-STEP PROCEDURE

<table>
<thead>
<tr>
<th>C#1</th>
<th>ON</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td></td>
<td>0.488</td>
<td>0.190</td>
<td>2.569</td>
<td>0.010</td>
</tr>
</tbody>
</table>
The latent class variable can be identified by any mixture model, not just LCA, for example Growth Mixture Models.

Multiple auxiliary variables can be analyzed at the same time.

Auxiliary variables can be included in a Monte Carlo setup.

The 3-step procedure can be setup manually for other types of models, different from the distal outcome model and the latent class regression. For example, distal outcomes regressed on the latent class variable and another predictor.
7.1 3-Step Mixture Modeling For Special Models: An Example

Bengt Muthén & Linda Muthén
How can we estimate a mixture regression model independently of the LCA model that defines $C$

$$Y = \alpha_c + \beta_c X + \varepsilon$$

We simulate data with $\alpha_1 = 0$, $\alpha_2 = 1$, $\beta_1 = 0.5$, $\beta_2 = -0.5$

Step 1: Estimate the LCA model (without the auxiliary model) with the following option
```
SAVEDATA: FILE=1.dat; SAVE=CPROB;
```

The above option creates the most likely class variable $N$

Step 2: Compute the error rate for $N$. In the LCA output find

```
Average Latent Class Probabilities for Most Likely Latent Class Membership by Latent Class (Column)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.835</td>
<td>0.165</td>
</tr>
<tr>
<td>2</td>
<td>0.105</td>
<td>0.895</td>
</tr>
</tbody>
</table>
```
3-Step Mixture Modeling For Special Models, Continued

- Compute the nominal variable $N$ parameters

\[
\log(0.835/0.165) = 1.621486
\]
\[
\log(0.105/0.895) = -2.14286
\]

- Step 3: estimate the model where $N$ is a latent class indicator with the above fixed parameters and include the class specific $Y$ on $X$ model

- When the class separation in the LCA is pretty good then $N$ is almost a perfect $C$ indicator
VARIABLE:  NAMES = u1-u5 y x p1 p2 n;
            NOMINAL = n;
            CLASSES = c(2);
            USEVARIABLES = y x n;

MODEL:

%OVERALL%
   y ON x;
%c#1%
   [n#1@1.621486];
   y ON x;
%c#2%
   [n#1@-2.14286];
   y ON x;
<table>
<thead>
<tr>
<th>Latent Class 1</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y ON X</td>
<td>0.546</td>
<td>0.054</td>
<td>10.185</td>
<td>0.000</td>
</tr>
<tr>
<td>Means N#1</td>
<td>1.621</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>Intercepts Y</td>
<td>0.101</td>
<td>0.052</td>
<td>1.961</td>
<td>0.050</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Latent Class 2</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y ON X</td>
<td>-0.483</td>
<td>0.051</td>
<td>-9.496</td>
<td>0.000</td>
</tr>
<tr>
<td>Means N#1</td>
<td>-2.143</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>Intercepts Y</td>
<td>1.037</td>
<td>0.056</td>
<td>18.498</td>
<td>0.000</td>
</tr>
</tbody>
</table>
7.2 Rules of Thumb: 1-Step Versus 3-Step Versus Most Likely Class Ignoring The Measurement Error

Consider a latent class model with covariates that is correctly specified. Choosing among 1-step, 3-step, and 3-step using Most likely class and ignoring misclassification error, the best approach to use depends to a large extent on the entropy (classification quality: 0-1):

- **Entropy < 0.6**: Use 1-step. 3-step and Most likely class don’t work well
- **0.6 < Entropy < 0.8**: 1-step and 3-step work well, but not Most likely class
- **Entropy > 0.8**: All three approaches work well

3-step needs large sample size \((n > 500)\); see Mplus Web Note 15.
### Transition Probabilities

<table>
<thead>
<tr>
<th></th>
<th>c2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Diagram**:

- **Time Point 1**:
  - u11
  - u12
  - u13
  - u14
- **Time Point 2**:
  - u21
  - u22
  - u23
  - u24
- **Latent Variables**:
  - c1
  - c2
Steps In Latent Transition Analysis

- Step 1: Study measurement model alternatives for each time point
- Step 2: Explore transitions based on cross-sectional results
  - Cross-tabs based on most likely class membership
- Step 3: Explore specification of the latent transition model without covariates
  - Testing for measurement invariance across time
- Step 4: Include covariates in the LTA model
- Step 5: Include distal outcomes and advanced modeling extensions

Source: Nylund (2007)

- Early Childhood Longitudinal Study - Kindergarten cohort
- Four time points: Kindergarten Fall, Spring and Grade 1 Fall, Spring; \( n = 3,575 \)
- Five dichotomous proficiency scores: Letter recognition, beginning sounds, ending letter sounds, sight words, words in context
- Binary poverty index
- LCA suggests 3 classes: Low alphabet knowledge (LAK), early word reading (EWR), and early reading comprehension (ERC)
Three latent classes:

- Class 1: Low alphabet knowledge (LAK)
- Class 2: Early word reading (EWR)
- Class 3: Early reading comprehension (ERC)

The ECLS-K LTA model has the special feature of specifying no decline in knowledge as zero transition probabilities. For example, transition from Kindergarten Fall to Spring:

LATENT TRANSITION PROBABILITIES
BASED ON THE ESTIMATED MODEL

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.655</td>
<td>0.017</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.646</td>
<td>0.354</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
LTA Example 1: ECLS-K.  
Transition Tables for the Binary Covariate Poverty

<table>
<thead>
<tr>
<th></th>
<th>Poverty = 0 (cp=1)</th>
<th>Poverty = 1 (cp=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c2</td>
<td>c2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>c1</td>
<td>1</td>
<td>0.252</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Bengt Muthén & Linda Muthén  
Mplus Modeling  
82/168

- Student’s self-reported peer victimization in Grade 6, 7, and 8
- Low SES, ethnically diverse public middle schools in the Los Angeles area (11% Caucasian, 17% Black, 48% Latino, 12% Asian)
- \( n = 2045 \)
- 6 binary items: Picked on, laughed at, called bad names, hit and pushed around, gossiped about, things taken or messed up (Neary & Joseph, 1994 Peer Victimization Scale)
LTA Example 2: Mover-Stayer Model

- Class 1: Victimized (G6-G8: 19%, 10%, 8%)
- Class 2: Sometimes victimized (G6-G8: 34%, 27%, 21%)
- Class 3: Non-victimized (G6-G8: 47%, 63%, 71%)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c2 (Grade 7)</td>
<td></td>
<td>c3 (Grade 8)</td>
</tr>
<tr>
<td>c1</td>
<td>0.29 0.45 0.26</td>
<td></td>
<td>0.23 0.59 0.18</td>
</tr>
<tr>
<td>(Grade 6)</td>
<td>0.06 0.44 0.51</td>
<td></td>
<td>0.04 0.47 0.49</td>
</tr>
<tr>
<td></td>
<td>0.04 0.46 0.55</td>
<td></td>
<td>0.06 0.17 0.77</td>
</tr>
<tr>
<td>c2 (Grade 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c3 (Grade 8)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c2 (Grade 7)</td>
<td></td>
<td>c3 (Grade 8)</td>
</tr>
<tr>
<td>c1</td>
<td>1 0 0</td>
<td></td>
<td>1 0 0</td>
</tr>
<tr>
<td>(Grade 6)</td>
<td>0 1 0</td>
<td></td>
<td>0 1 0</td>
</tr>
<tr>
<td></td>
<td>0 0 1</td>
<td></td>
<td>0 0 1</td>
</tr>
</tbody>
</table>
Consider the logit parameterization for CLASSES = c1(3) c2(3):

<table>
<thead>
<tr>
<th>c1</th>
<th>c2</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a1 + b11</td>
<td>a2 + b21</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a1 + b12</td>
<td>a2 + b22</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a1</td>
<td>a2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

where each row shows the logit coefficients for a multinomial logistic regression of c2 on c1 with the last c2 class as reference class.

Zero lower-triangular probabilities are obtained by fixing the a1, a2, and b12 parameters at the logit value -15. The parameters b11, b21, and b22 are estimated.
TITLE: LTA of Kindergarten Fall and Spring (3 x 3)
DATA: FILE = dp.analytic.dat;
       FORMAT = f1.0, 20f2.0;
VARIABLE: NAMES = pov letrec1 begin1 ending1 sight1 wic1 
               letrec2 begin2 ending2 sight2 wic2 
               letrec3 begin3 ending3 sight3 wic3 
               letrec4 begin4 ending4 sight4 wic4;
USEVARIABLES = letrec1 begin1 ending1 sight1 wic1 
               letrec2 begin2 ending2 sight2 wic2;
               ! letrec3 begin3 ending3 sight3 wic3 
               ! letrec4 begin4 ending4 sight4 wic4;
CATEGORICAL = letrec1 begin1 ending1 sight1 wic1 
               letrec2 begin2 ending2 sight2 wic2;
               ! letrec3 begin3 ending3 sight3 wic3 
               ! letrec4 begin4 ending4 sight4 wic4;
CLASSES = c1(3) c2(3);
MISSING = .;
ANALYSIS:  TYPE = MIXTURE;
STARTS = 400 80;
PROCESSORS = 8;
MODEL:  %OVERALL%
! fix lower triangular transition probabilities = 0:
[c2#1@-15 c2#2@-15]; ! fix a1 = a2 = -15
c2#1 ON c1#2@-15; ! fix b12 = -15
c2#1 ON c1#1*15; ! b11: start at 15 to make total logit start=0
c2#2 ON c1#1-c1#2*15; ! b21, b22
MODEL c1:
  %c1#1%
  [letrec1$1-wic1$1] (1-5);
  %c1#2%
  [letrec1$1-wic1$1] (6-10);
  %c1#3%
  [letrec1$1-wic1$1] (11-15);

MODEL c2:
  %c2#1%
  [letrec2$1-wic2$1] (1-5);
  %c2#2%
  [letrec2$1-wic2$1] (6-10);
  %c2#3%
  [letrec2$1-wic2$1] (11-15);

OUTPUT: TECH1 TECH15;
PLOT: TYPE = PLOT3;
       SERIES = letrec1-wic1(*) | letrec2-wic2(*);
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.655</td>
<td>0.017</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.646</td>
<td>0.354</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**LATENT TRANSITION PROBABILITIES**

**BASED ON THE ESTIMATED MODEL**

c₁ classes (rows) by c₂ classes (columns)
• TECH15 output with conditional class probabilities useful for studying transition probabilities with an observed binary covariate such as treatment/control or a latent class covariate
• LTA transition probability calculator for continuous covariates
• Probability parameterization to simplify input for Mover-Stayer LTA and other models with restrictions on the transition probabilities
CLASSES = cp(2) c1(3) c2(3);
KNOWNCLASS = cp(pov=0 pov=1);

ANALYSIS: TYPE = MIXTURE;
STARTS = 400 80; PROCESSORS = 8;

MODEL:
%OVERALL%
c1 ON cp;
[c2#1@-15 c2#2@-15];
c2#1 ON c1#2@-15;

MODEL cp: %cp#1%
c2#1 ON c1#1*15;
c2#2 ON c1#1-c1#2*15;
%c2#2%
c2#1 ON c1#1*15;
c2#2 ON c1#1-c1#2*15;

MODEL c1: %c1#1% etc as before
P(CP=1) = 0.808
P(CP=2) = 0.192

P(C1=1|CP=1) = 0.617
P(C1=2|CP=1) = 0.351
P(C1=3|CP=1) = 0.032

P(C1=1|CP=2) = 0.872
P(C1=2|CP=2) = 0.123
P(C1=3|CP=2) = 0.005

P(C2=1|CP=1,C1=1) = 0.252
P(C2=2|CP=1,C1=1) = 0.732
P(C2=3|CP=1,C1=1) = 0.017

P(C2=1|CP=1,C1=2) = 0.000
P(C2=2|CP=1,C1=2) = 0.647
P(C2=3|CP=1,C1=2) = 0.353
P(C2=1|CP=1,C1=3)=0.000
P(C2=2|CP=1,C1=3)=0.000
P(C2=3|CP=1,C1=3)=1.000

P(C2=1|CP=2,C1=1)=0.545
P(C2=2|CP=2,C1=1)=0.442
P(C2=3|CP=2,C1=1)=0.013

P(C2=1|CP=2,C1=2)=0.000
P(C2=2|CP=2,C1=2)=0.620
P(C2=3|CP=2,C1=2)=0.380

P(C2=1|CP=2,C1=3)=0.000
P(C2=2|CP=2,C1=3)=0.000
P(C2=3|CP=2,C1=3)=1.000

New feature in Version 7: The LTA calculator
Interaction Displayed Two Equivalent Ways

\[ x \]

\[ \text{c1} \]

\[ \text{c2} \]

- \text{u11}
- \text{u12}
- \text{u13}
- \text{u14}
- \text{u21}
- \text{u22}
- \text{u23}
- \text{u24}

\[ \text{c1} \]

\[ \text{c2} \]

- \text{u11}
- \text{u12}
- \text{u13}
- \text{u14}
- \text{u21}
- \text{u22}
- \text{u23}
- \text{u24}

\[ x \]
Review of Logit Parameterization with Covariates: Parameterization 2

\[
\begin{align*}
\text{c}_1 & \quad \text{c}_2 \\
1 & \quad a_1 + b_{11} + g_{11} x \\n2 & \quad a_1 + b_{12} + g_{12} x \\n3 & \quad a_1 + g_{13} x \\
\text{c}_2 & \\
1 & \quad a_2 + b_{21} + g_{21} x \\n2 & \quad a_2 + b_{22} + g_{22} x \\n3 & \quad a_2 + g_{23} x
\end{align*}
\]

MODEL: %OVERALL%
\[
c_1 \text{ ON } x;
\]
\[
c_2 \text{ ON } c_1;
\]

MODEL c1: %c1#1%
\[
c_2#1 \text{ ON } x (g_{11});
\]
\[
c_2#2 \text{ ON } x (g_{21});
\]

%c1#2%
\[
c_2#1 \text{ ON } x (g_{12});
\]
\[
c_2#2 \text{ ON } x (g_{22});
\]

%c1#3%
\[
c_2#1 \text{ ON } x (g_{13});
\]
\[
c_2#2 \text{ ON } x (g_{23});
\]
USEVARIABLES = letrec1 begin1 ending1 sight1 wic1 
letrec2 begin2 ending2 sight2 wic2 pov; 
! letrec3 begin3 ending3 sight3 wic3 
! letrec4 begin4 ending4 sight4 wic4;  
CATEGORICAL = letrec1 begin1 ending1 sight1 wic1 
letrec2 begin2 ending2 sight2 wic2; 
! letrec3 begin3 ending3 sight3 wic3 
! letrec4 begin4 ending4 sight4 wic4;  
CLASSES = c1(3) c2(3);  
MISSING = .;  
ANALYSIS: TYPE = MIXTURE; 
STARTS = 400 80; 
PROCESSORS = 8;
MODEL:  %OVERALL%
c1 ON pov;
! do c2 ON pov in c1-specific model part to get interaction
[c2#1@-15 c2#2@-15]; ! to give zero probability of declining
(c2#1 ON c1#2@-15); ! to give zero probability of declining
(c2#1 ON c1#1*15);
c2#2 ON c1#1-c1#2*15;

MODEL c1:  %c1#1%
c2 ON pov; ! (g11) and (g21)
%c1#2%
c2#1 ON pov@-15; ! to give zero probability of declining (g12)
c2#2 ON pov; ! (g22)

! %c1#3% not mentioned due to g13=0, g23=0 by default
LTA Calculator Applied to Poverty
Estimated conditional probabilities for the latent class variables:

<table>
<thead>
<tr>
<th>Condition(s):</th>
<th>POV = 1.000000</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(C1=1)=0.872</td>
<td>P(C2=1</td>
</tr>
<tr>
<td>P(C1=2)=0.123</td>
<td>P(C2=2</td>
</tr>
<tr>
<td>P(C1=3)=0.005</td>
<td>P(C2=3</td>
</tr>
<tr>
<td></td>
<td>P(C2=1</td>
</tr>
<tr>
<td></td>
<td>P(C2=2</td>
</tr>
<tr>
<td></td>
<td>P(C2=3</td>
</tr>
<tr>
<td></td>
<td>P(C2=1</td>
</tr>
<tr>
<td></td>
<td>P(C2=2</td>
</tr>
<tr>
<td></td>
<td>P(C2=3</td>
</tr>
</tbody>
</table>
8.3.3 Probability Parameterization

- New feature in Mplus Version 7
- LTA models that do not have continuous x’s can be more conveniently specified using PARAMETERIZATION=PROBABILITY to reflect hypotheses expressed in terms of probabilities
- Useful for Mover-Stayer LTA models
Probability parameterization for CLASSES = c1(3) c2(3):

<table>
<thead>
<tr>
<th>c1</th>
<th>c2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>p11</td>
<td>p12</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>p21</td>
<td>p22</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>p31</td>
<td>p32</td>
<td>0</td>
</tr>
</tbody>
</table>

where the probabilities in each row add to 1 and the last c2 class is not mentioned. The p parameters are referred to using ON. The latent class variable c1 which is the predictor has probability parameters [c1#1 c1#2], whereas "intercept" parameters are not included for c2.

A transition probability can be conveniently fixed at 1 or 0 by using the p parameters.
8.3.4 Mover-Stayer LTA in Probability Parameterization

![Diagram of Mover-Stayer LTA in Probability Parameterization](image)
ANALYSIS: PARAMETERIZATION = PROBABILITY;
MODEL: %OVERALL% ! Relating c1 to c:
c1 ON c;
MODEL c: %c#1% ! Mover class
c2 ON c1;
c3 ON c2;
%c#2% ! Stayer class
c2#1 ON c1#1@1; c2#2 ON c1#1@0;
c2#1 ON c1#2@0; c2#2 ON c1#2@1;
c2#1 ON c1#3@0; c2#2 ON c1#3@0;
c3#1 ON c2#1@1; c3#2 ON c2#1@0;
c3#1 ON c2#2@0; c3#2 ON c2#2@1;
c3#1 ON c2#3@0; c3#2 ON c2#3@0;
!measurement part as before
Mover-Stayer LTA in Probability Parameterization: Predicting Mover-Stayer Class Membership From A Nominal Covariate
VARIABLE:  
CLASSES = cg(5) c (2) c1(3) c2(3) c3(3);
KNOWNCLASS = cg(eth=0 eth=1 eth=2 eth=3 eth=4);
ANALYSIS:  TYPE = MIXTURE COMPLEX;
STARTS = 400 100;
PROCESS = 8;
PARAMETERIZATION = PROBABILITY;
MODEL:  
%OVERALL%
c ON cg#1-cg#5 (b1-b5);
c1 ON c;
MODEL c: etc
MODEL CONSTRAINT:
NEW(logor2-logor5);
  ! log of ratio of odds of being Mover vs Stayer for the groups
logor2 = log(((b2/(1-b2))/(b1/(1-b1)))); ! eth=1 (cg=2) vs 0 (cg=1)
logor3 = log(((b3/(1-b3))/(b1/(1-b1)))); ! eth=2 (cg=3) vs 0
logor4 = log(((b4/(1-b4))/(b1/(1-b1)))); ! eth=3 (cg=4) vs 0
logor5 = log(((b5/(1-b5))/(b1/(1-b1)))); ! eth=4 (cg=5) vs 0
8.4 Latent Transition Analysis Extensions:
Factor Mixture Latent Transition Analysis Muthén (2006)
1,137 first-grade students in Baltimore public schools

9 items: Stubborn, Break rules, Break things, Yells at others, Takes others property, Fights, Lies, Teases classmates, Talks back to adults

Skewed, 6-category items; dichotomized (almost never vs other)

Two time points: Fall and Spring of Grade 1

For each time point, a 2-class, 1-factor FMA was found best fitting
### Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

<table>
<thead>
<tr>
<th>Model</th>
<th>Loglikelihood</th>
<th># parameters</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>conventional LTA</td>
<td>-8,649</td>
<td>21</td>
<td>17,445</td>
</tr>
<tr>
<td>LTA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FMA LTA factors related across time</td>
<td>-8,102</td>
<td>40</td>
<td>16,306</td>
</tr>
</tbody>
</table>
Estimated latent transition probabilities, fall to spring

<table>
<thead>
<tr>
<th></th>
<th>conventional LTA</th>
<th>FMA-LTA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>low</td>
<td>0.93</td>
<td>0.07</td>
</tr>
<tr>
<td>high</td>
<td>0.17</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Example: Number of parameters for 11 $u$’s and 3 classes:

<table>
<thead>
<tr>
<th></th>
<th>LCA</th>
<th>LCGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>binary $u$</td>
<td>35</td>
<td>11</td>
</tr>
<tr>
<td>3-categ. $u$</td>
<td>68</td>
<td>12</td>
</tr>
</tbody>
</table>
9.1 Single Process Latent Class Growth Analysis: Cambridge Delinquency Data

- 411 boys in a working class section of London (n = 403 due to 8 boys who died)
- Ages 10 to 32 (ages 11 - 21 used here)
- Outcome is number of convictions in the last 2 years, modeled as an ordered polytomous variable scored 0 for 0 convictions, 1 for one conviction, and 2 for more than one conviction

Sources: Farrington & West (1990); Nagin & Land (1993); Roeder, Lynch & Nagin (1999); Muthén (2004)
Latent Class Analysis With 3 Classes On Cambridge Data

LogL = -1,032 (68 parameters), BIC = 2,472
<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE:</td>
<td>LCGA ordered polytomous variables for conviction at each age 11-21 dep. variable 0, 1, 2 (0, 1, or more convictions)</td>
</tr>
<tr>
<td>DATA:</td>
<td>FILE = naginordered.dat;</td>
</tr>
<tr>
<td>VARIABLE:</td>
<td>NAMES = u11 u12 u13 u14 u15 u16 u17 u18 u19 u20 u21 c1 c2 c3 c4;</td>
</tr>
<tr>
<td></td>
<td>USEVAR = u11-u21;</td>
</tr>
<tr>
<td></td>
<td>CATEGORICAL = u11-u21;</td>
</tr>
<tr>
<td></td>
<td>CLASSES = c(3);</td>
</tr>
<tr>
<td>ANALYSIS:</td>
<td>TYPE = MIXTURE;</td>
</tr>
<tr>
<td>MODEL:</td>
<td>%OVERALL% i s q</td>
</tr>
<tr>
<td>OUTPUT:</td>
<td>TECH1 TECH8;</td>
</tr>
<tr>
<td>PLOT:</td>
<td>SERIES = u11-u21(s);</td>
</tr>
<tr>
<td></td>
<td>TYPE = PLOT3;</td>
</tr>
</tbody>
</table>
3-class LCGA
- LogL = -1,072
- 12 parameters
- BIC = 2,215

3-class LCA
- LogL = -1,032
- (68 parameters)
- BIC = 2,472
Co-Occurrence Of Alcohol And Tobacco Use Disorder

Class-Probability Estimates

<table>
<thead>
<tr>
<th>TD</th>
<th>low</th>
<th>up</th>
<th>chronic</th>
<th>chronic</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>.61</td>
<td>.08</td>
<td>.001</td>
<td>.69</td>
</tr>
<tr>
<td>AUD</td>
<td>.15</td>
<td>.07</td>
<td>.03</td>
<td>.25</td>
</tr>
<tr>
<td>down</td>
<td>.04</td>
<td>.02</td>
<td>.003</td>
<td>.06</td>
</tr>
<tr>
<td>chron</td>
<td>.80</td>
<td>.17</td>
<td>.03</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Are All Individuals From The Same Population?

\[ \begin{align*}
(1) \quad y_{ti} & = i_i + s_i \ \text{time}_{ti} + \varepsilon_{ti} \\
(2a) \quad i_i & = \alpha_0 + \gamma_0 \ w_i + \zeta_0 i \\
(2b) \quad s_i & = \alpha_1 + \gamma_1 \ w_i + \zeta_1 i
\end{align*} \]
Mixtures And Latent Trajectory Classes

Modeling motivated by substantive theories of:

- **Multiple Disease Processes**: Prostate cancer (Pearson et al.)
- **Multiple Pathways of Development**: Adolescent-limited versus life-course persistent antisocial behavior (Moffitt), crime curves (Nagin), alcohol development (Zucker, Schulenberg)
- **Different response to medication such as placebo response to antidepressants**: (Muthén & Brown, 2009, Statistics in Medicine; Muthén et al., 2011, APPA book)
Example: Mixed-Effects Regression Models For Studying The Natural History Of Prostate Disease

Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)

Source: Pearson, Morrell, Landis & Carter (1994), Statistics in Medicine
A Clinical Trial Of Antidepressants Growth Mixture Modeling With Placebo Response (Muthén et al, 2011)
A Clinical Trial Of Antidepressants Growth Mixture Modeling With Placebo Response (Muthén et al, 2011)

Responder Class

Non-Responder Class

Bengt Muthén & Linda Muthén

Mplus Modeling
Growth Modeling Paradigms

HLM (Raudenbush)  GMM (Muthén)  LCGA (Nagin)

- Replace rhetoric with statistics – let likelihood decide
- HLM and LCGA are special cases of GMM
10.1 Philadelphia Crime Data: ZIP Growth Mixture Modeling

- 13,160 males ages 4 - 26 born in 1958 (Moffitt, 1993; Nagin & Land, 1993)
- Annual counts of police contacts
- Individuals with more than 10 counts in any given year deleted (n=13,126)
- Data combined into two-year intervals
**Zero-Inflated Poisson (ZIP)**

**Growth Mixture Modeling Of Counts**

\[

u_{ti} = \begin{cases} 
0 & \text{with probability } \pi_{ti} \\
\text{Poisson}(\lambda_{ti}) & \text{with probability } 1 - \pi_{ti}
\end{cases} \tag{1}

\ln \lambda_{ti|C_i=c} = \eta_{0i} + \eta_{1i} \alpha_{ti} + \eta_{2i} \alpha_{ti}^2 \tag{2}

\eta_{0i|C_i=c} = \alpha_0 c + \zeta_{0i} \tag{3}

\eta_{1i|C_i=c} = \alpha_1 c + \zeta_{1i} \tag{4}

\eta_{2i|C_i=c} = \alpha_2 c + \zeta_{2i} \tag{5}

\]

In Mplus, \( \pi_{ti} = P(u\#_{ti} = 1) \), where \( u\# \) is a binary latent inflation variable and \( u\# = 1 \) indicates that the individual is unable to assume any value except 0.
<table>
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<tr>
<th>Model</th>
<th>Log-Likelihood</th>
<th># Parameters</th>
<th>BIC</th>
<th># Significant Residuals</th>
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<td>-40,263</td>
<td>39</td>
<td>80,896</td>
<td>1</td>
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</tbody>
</table>
Three-Class ZIP GMM
For Philadelphia Crime
VARIABLE: USEVAR = y10 y12 y14 y16 y18 y20 y22 y24;
!y10 = ages 10-11, y12 = ages y12-13, etc
IDVAR = cohortid;
USEOBS = y10 LE 10 AND y12 LE 10 AND y14 LE 10 AND
y16 LE 10 AND y18 LE 10 AND y20 LE 10 AND y22 LE 10
AND y24 LE 10;
COUNT = y10-y24(i);
CLASSES = c(3);

ANALYSIS: TYPE = MIXTURE;
ALGORITHM = INTEGRATION;
PROCESS = 8;
INTEGRATION = 10;
STARTS = 50 5;
INTERACTIVE = control.dat;

MODEL:
  %OVERALL%
  i s q | y10@0 y12@.1 y14@.2 y16@.3 y18@.4 y20@.5 y22@.6 y24@.7;

OUTPUT:  TECH1 TECH10;
PLOT:     TYPE = PLOT3;
          SERIES = y10-y24(s);
10.2 LSAy Math Achievement Trajectory Classes

Poor Development: 20%

Moderate Development: 28%

Good Development: 52%

Dropout: 69% 8% 1%
VARIABLE:  USEVARIABLES = female mothed homeres math7 math8 math9 math10 expel arrest hisp black hsdrop expect droptht7;
CATEGORICAL = hsdrop;
CLASSES = c(3);
ANALYSIS:  TYPE = MIXTURE; STARTS = 0;
MODEL:  %OVERALL%
i s | math7@0 math8@1 math9@2 math10@3;
i s ON mothed homeres expect droptht7 expel arrest female hisp black;
c ON mothed homeres expect droptht7 expel arrest female hisp black;
hsdrop ON mothed homeres expect droptht7 expel arrest female hisp black;
%c#1%
[i*36]; [s*0]; [hsdrop$1*-1]; ! to get the low class first
OUTPUT:  SAMPSTAT STANDARDIZED TECH1 TECH8;
PLOT:  TYPE = PLOT3;
<table>
<thead>
<tr>
<th>Categorical Latent Variables</th>
<th>Estimates</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>Two-Tailed P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>c#1 ON</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>moth ed</td>
<td>-0.251</td>
<td>0.122</td>
<td>-2.055</td>
<td>0.040</td>
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<tr>
<td>homer es</td>
<td>-0.240</td>
<td>0.114</td>
<td>-2.111</td>
<td>0.035</td>
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<tr>
<td>expect</td>
<td>-0.512</td>
<td>0.183</td>
<td>-2.805</td>
<td>0.005</td>
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<tr>
<td>droptht7</td>
<td>1.267</td>
<td>0.659</td>
<td>1.922</td>
<td>0.055</td>
</tr>
<tr>
<td>expel</td>
<td>1.903</td>
<td>0.526</td>
<td>3.616</td>
<td>0.000</td>
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<tr>
<td>arrest</td>
<td>0.385</td>
<td>0.485</td>
<td>0.795</td>
<td>0.427</td>
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<tr>
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<tr>
<td>hisp</td>
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<td>1.760</td>
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<td>0.380</td>
<td>66.780</td>
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<tr>
<td>-----------</td>
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<td>-4.123</td>
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<td>Est./S.E.</td>
<td>Two-Tailed P-Value</td>
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<td>--------------------</td>
<td>-----------</td>
<td>------</td>
<td>-----------</td>
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</tr>
<tr>
<td>hsdrop$1</td>
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<td>0.305</td>
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</tbody>
</table>

\[
P(hsdrop=1 \mid x=0) = \frac{1}{1+\exp(\text{threshold})}.\]
New setting:
- Sequential, linked processes

New aims:
- Using an earlier process to predict a later process
- Early prediction of failing class

Application: General growth mixture modeling of first- and second-grade reading skills and their Kindergarten precursors; prediction of reading failure (Muthén, Khoo, Francis, Boscardin, 1999). Suburban sample, $n = 410$. 
Longitudinal multiple-cohort design involving approximately 1000 children with measurements taken four times a year from Kindergarten through grade two (October, December, February, April)

- Grade 1 - Grade 2: reading and spelling skills
- Precursor skills: phonemic awareness (Kindergarten, Grade 1, Grade 2), letters/names/sounds (Kindergarten only), rapid naming
- Standardized reading comprehension tests at the end of Grade 1 and Grade 2 (May).
Three research hypotheses (EARS study; Francis, 1996):

- Kindergarten children will differ in their growth and development in precursor skills.

- The rate of development of the precursor skills will relate to the rate of development and the level of attainment of reading and spelling skills - and the individual growth rates in reading and spelling skills will predict performance on standardized tests of reading and spelling.

- The use of growth rates for skills and precursors will allow for earlier identification of children at risk for poor academic outcomes and lead to more stable predictions regarding future academic performance.
### Title: Growth mixture model for reading skills development

#### Data:
```
FILE = newran.dat;
```

#### Variable:
```
VARIABLE: NAMES = gender eth wc pa1-pa4 wr1-wr8 l1-l4 s1 r1 s2 r2 rnaming1 rnaming2 rnaming3 rnaming4;
USEVAR = pa1-wr8 rnaming4;
MISSING ARE ALL (999);
CLASSES = c(5);
```

#### Analysis:
```
ANALYSIS: TYPE = MIXTURE;
```

#### Model:
```
%OVERALL%
  i1 s1 | pa1@-3 pa2@-2 pa3@-1 pa4@0;
  i2 s2 | wr1@-7 wr2@-6 wr3@-5 wr4@-4 wr5@-3 wr6@-2 wr7@-1 wr8@0;
  c#1-c#4 ON rnaming4;
```

#### Output:
```
OUTPUT: TECH8;
```
Five Classes Of Reading Skills Development

Kindergarten Growth (Five Classes)
Phonemic Awareness

Grades 1 and 2 Growth (Five Classes)
Word Recognition
Focus on Class 1, the failing class.

1. Estimate full growth mixture model for Kindergarten, Grade 1, and Grade 2 outcomes.

2. Use the estimated full model to classify students into classes based on the posterior probabilities for each class, where a student is classified into the class with the largest posterior probability.

3. Classify students using early information by holding parameters fixed at the estimates from the full model of Step 1 and classifying individuals using Kindergarten information only, adding Grade 1 outcomes, adding Grade 2 outcomes.

4. Study quality of early classification by cross-tabulating individuals classified as in Steps 2 and 3 (sensitivity and specificity).
### Sensitivity And Specificity Of Early Classification

#### Full Model

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<td>71</td>
<td>144</td>
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### Full Model

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### Full Model

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<td>70</td>
<td>144</td>
<td>100</td>
<td>49</td>
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</tbody>
</table>
Different treatment effects in different trajectory classes


Muthén & Brown (2009), Statistics in Medicine: A sizable portion of responders in antidepressant trials may be placebo responders

See also Muthén & Curran, 1997 for monotonic treatment effects
GMM: treatment changes trajectory shape.
Modeling Treatment Effects

3-Class Model 1

- High Class, 14%
- Medium Class, 50%
- Low Class, 36%

Control
Intervention

BIC=9421
Entropy=0.83

Grades 1 - 7

1 2 3 4 5 6

1F 1S 2F 2S 3S 4S 5S 6S 7S

4-Class Model 1

- High Class, 15%
- Medium Class, 44%
- Low Class, 19%
- LS Class, 22%

Control
Intervention

BIC=8394
Entropy=0.80

Grades 1 - 7

1 2 3 4 5 6

1F 1S 2F 2S 3S 4S 5S 6S 7S
11. Data Not Missing At Random: Non-Ignorable Dropout In Longitudinal Studies

- Missing completely at random (MCAR)
- Missing at random (MAR)
- Not missing at random (NMAR)
  - Selection modeling
  - Pattern-mixture modeling
  - General latent variable modeling
Subjects treated with citalopram (Level 1). No placebo group
Sample means of the QIDS depression score at each visit:

Next level

Follow-up
Growth Mixture Model Assuming MAR

\[ \text{c} \rightarrow \text{i} \rightarrow \text{s} \rightarrow \text{q} \]

\[ \text{y}_0 \rightarrow \text{y}_1 \rightarrow \text{y}_2 \rightarrow \text{y}_3 \rightarrow \text{y}_4 \rightarrow \text{y}_5 \]
Not Missing At Random (NMAR): Non-Ignorable Dropout Modeling

NMAR: Missingness influenced by latent variables

- Data to be modeled are not only outcomes but also missing data indicators
- Two general approaches:
  - Selection modeling: Growth features influence dropout occasion
  - Pattern-mixture modeling: Dropout occasion influences growth parameters

Beunckens Mixture Model (Mixture Wu-Carroll Model): Adding Dropout Information (Survival Indicators)
4-class Beunckens Selection Mixture Model
Muthén-Roy Pattern-Mixture Model (d’s are dropout dummies)
The NMAR approach of adding dropout information gives a less favorable conclusion regarding drug response than the standard assumption of MAR.

<table>
<thead>
<tr>
<th>Model</th>
<th>Response class</th>
<th>Temporary response class</th>
<th>Non-response class</th>
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<tbody>
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<td>55 %</td>
<td>3 %</td>
<td>15 %</td>
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<td>NMAR models:</td>
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</tr>
<tr>
<td>Beuncken</td>
<td>35 %</td>
<td>19 %</td>
<td>25 %</td>
</tr>
<tr>
<td>Muthén-Roy</td>
<td>32 %</td>
<td>15 %</td>
<td>14 %</td>
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</tbody>
</table>
12. Survival Modeling
With Continuous and Categorical Latent Variables


- Larsen (2005). The Cox proportional hazards model with a continuous latent variable measured by multiple binary indicators. *Biometrics*


12.1 Cancer Survival Trial of Second-Line Treatment of Mesothelioma

Kaplan-Meier Curves

Control
Treatment

Progression-Free Survival (months)
Probability
### Patient-Reported Lung Cancer Symptom Scale (LCSS)

**Directions:** Please place a mark along each line where it would best describe the symptoms of your lung illness DURING THE PAST DAY (during the past 24 hours)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How is your appetite?</td>
<td>As good as it could be</td>
</tr>
<tr>
<td>2. How much fatigue do you have?</td>
<td>None</td>
</tr>
<tr>
<td>3. How much coughing do you have?</td>
<td>None</td>
</tr>
<tr>
<td>4. How much shortness of breath do you have?</td>
<td>None</td>
</tr>
<tr>
<td>5. How much blood do you see in your sputum?</td>
<td>None</td>
</tr>
<tr>
<td>6. How much pain do you have?</td>
<td>None</td>
</tr>
<tr>
<td>7. How bad are your symptoms from your lung illness?</td>
<td>I have none</td>
</tr>
<tr>
<td>8. How much has your illness affected your ability to carry out normal activities?</td>
<td>Not at all</td>
</tr>
<tr>
<td>9. How would you rate the quality of your life today?</td>
<td>Very high</td>
</tr>
</tbody>
</table>
Predicting Survival From Visit 0 Using a Factor Mixture Model For LCSS Items
Survival Curves Showing Overall Treatment Effect

Kaplan-Meier Curves

- Red line: Control
- Blue line: Treatment

Probability vs. Progression-Free Survival (months)
Survival Curves For Low-Symptom Class

Estimated Survival Curves For Factor Mixture Model, Class 1, stage=5, prior=1, kps0=mean

- Red line: Control
- Blue line: Treatment

Probability vs. Progression-Free Survival (months)
For references, see handouts for Topics 1 - 9 at
http://www.statmodel.com/course_materials.shtml

For handouts and videos of Version 7 training, see
http://mplus.fss.uu.nl/2012/09/12/the-workshop-new-features-of-mplus-v7/

For papers using special Mplus features, see
http://www.statmodel.com/papers.shtml