Mplus Short Courses
Topic 7

Multilevel Modeling With Latent Variables Using Mplus: Cross-Sectional Analysis

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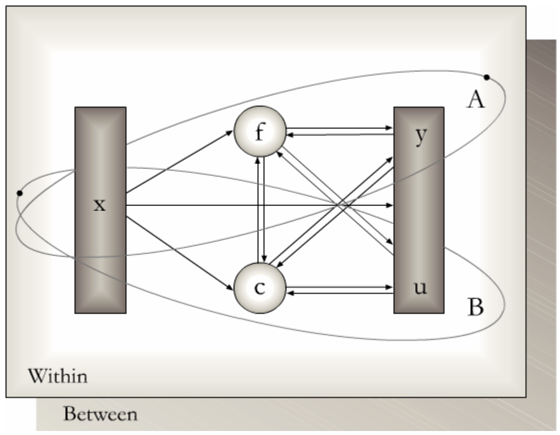
Mplus Background

- Inefficient dissemination of statistical methods:
  - Many good methods contributions from biostatistics, psychometrics, etc are underutilized in practice
- Fragmented presentation of methods:
  - Technical descriptions in many different journals
  - Many different pieces of limited software
- Mplus: Integration of methods in one framework
  - Easy to use: Simple, non-technical language, graphics
  - Powerful: General modeling capabilities
- Mplus versions
  - V1: November 1998
  - V2: February 2001
  - V3: March 2004
  - V4: February 2006
  - V5: November 2007
  - V5.2: November 2008
- Mplus team: Linda & Bengt Muthén, Thuy Nguyen, Tihomir Asparouhov, Michelle Conn, Jean Maninger

General Latent Variable Modeling Framework



Mplus

Several programs in one

- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Monte Carlo simulation

Fully integrated in the general latent variable framework

Overview Of Mplus Courses

- **Topic 1.** March 18, 2008, Johns Hopkins University: Introductory - advanced factor analysis and structural equation modeling with continuous outcomes
- **Topic 2.** March 19, 2008, Johns Hopkins University: Introductory - advanced regression analysis, IRT, factor analysis and structural equation modeling with categorical, censored, and count outcomes

- **Topic 3.** August 21, 2008, Johns Hopkins University: Introductory and intermediate growth modeling
- **Topic 4.** August 22, 2008, Johns Hopkins University: Advanced growth modeling, survival analysis, and missing data analysis

Overview Of Mplus Courses (Continued)

- **Topic 5.** November 10, 2008, University of Michigan, Ann Arbor: Categorical latent variable modeling with cross-sectional data
- **Topic 6.** November 11, 2008, University of Michigan, Ann Arbor: Categorical latent variable modeling with longitudinal data

- **Topic 7.** March 17, 2009, Johns Hopkins University: Multilevel modeling of cross-sectional data
- **Topic 8.** March 18, 2009, Johns Hopkins University: Multilevel modeling of longitudinal data

Analysis With Multilevel Data

Analysis With Multilevel Data

Used when data have been obtained by cluster sampling and/or unequal probability sampling to avoid biases in parameter estimates, standard errors, and tests of model fit and to learn about both within- and between-cluster relationships.

Analysis Considerations

- Sampling perspective
  - Aggregated modeling – SUDAAN
    - TYPE = COMPLEX
      - Clustering, sampling weights, stratification (Asparouhov, 2005)

Analysis With Multilevel Data (Continued)

- Multilevel perspective
  - Disaggregated modeling – multilevel modeling
    - TYPE = TWOLEVEL
      - Clustering, sampling weights, stratification
  - Multivariate modeling
    - TYPE = GENERAL
      - Clustering, sampling weights
- Combined sampling and multilevel perspective
  - TYPE = COMPLEX TWOLEVEL
    - Clustering, sampling weights, stratification

Analysis With Multilevel Data (Continued)

Analysis Areas

- Multilevel regression analysis
- Multilevel path analysis
- Multilevel factor analysis
- Multilevel SEM
- Multilevel growth modeling
- Multilevel latent class analysis
- Multilevel latent transition analysis
- Multilevel growth mixture modeling

Complex Survey Data Analysis

Intraclass Correlation

Consider nested, random-effects ANOVA for unit i in cluster j,

$$y_{ij} = \nu + \eta_j + \varepsilon_{ij}\ ;\ i = 1, 2,\dots, n_j\ ;\ j = 1,2,\dots, J. \qquad (44)$$

Random sample of J clusters (e.g. schools).

With timepoint as i and individual as j, this is a repeated measures model with random intercepts.

Consider the covariance and variances for cluster members $i = k$ and $i = l$,

$$Cov(y_{kj}, y_{lj}) = V(\eta), \qquad (45)$$

$$V(y_{kj}) = V(y_{lj}) = V(\eta) + V(\varepsilon), \qquad (46)$$

resulting in the intraclass correlation

$$\rho(y_{kj}, y_{lj}) = V(\eta)/[V(\eta) + V(\varepsilon)]. \qquad (47)$$

Interpretation: Between-cluster variability relative to total variation, intra-cluster homogeneity.

NLSY Household Clusters

Household Type (# of respondents)	# of Households*	Intraclass Correlations for Siblings Year	Heavy Drinking
Single	5,944	1982	0.19
Two	1,985	1983	0.18
Three	634	1984	0.12
Four	170	1985	0.09
Five	32	1988	0.04
Six	5	1989	0.06

Total number of households: 8,770

Total number of respondents: 12,686

Average number of respondents per household: 1.4

*Source: NLS User's Guide, 1994, p.247

Design Effects

Consider cluster sampling with equal cluster sizes and the sampling variance of the mean.

V_C : correct variance under cluster sampling
V_{SRS} : variance assuming simple random sampling

$V_C \geq V_{SRS}$ but cluster sampling more convenient, less expensive.

$$DEFF = V_C / V_{SRS} = 1 + (s - 1)\,\rho, \qquad (47)$$

where s is the common cluster size and ρ is the intraclass correlation (common range: 0.00 – 0.50).

Random Effects ANOVA Example

200 clusters of size 10 with intraclass correlation 0.2 analyzed as:

- TYPE = TWOLEVEL
- TYPE = COMPLEX
- Regular analysis, ignoring clustering

  DEFF = 1 + 9 * 0.2 = 2.8

Input For Two-Level Random Effects ANOVA Analysis

```
TITLE:      Random effects ANOVA data
            Two-level analysis with balanced data

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
            CLUSTER = cluster;

ANALYSIS:   TYPE = TWOLEVEL;

MODEL:
            %WITHIN%
            y;
            %BETWEEN%
            y;
```

Output Excerpts Two-Level Random Effects ANOVA Analysis

Model Results

```
                    Estimates      S.E.       Est./S.E.
Within Level
  Variances
     Y                  0.779       0.025         31.293
Between Level
  Means
     Y                  0.003       0.038          0.076
  Variances
     Y                  0.212       0.028          7.496
```


Input For Complex Random Effects ANOVA Analysis

```
TITLE:      Random effects ANOVA data
            Complex analysis with balanced data

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
            CLUSTER = cluster;

ANALYSIS:   TYPE = COMPLEX;
```

Output Excerpts Complex Random Effects ANOVA Analysis

Model Results

```
                    Estimates      S.E.       Est./S.E.
  Means
     Y                  0.003      0.038          0.076
  Variances
     Y                  0.990      0.036         27.538
```


Input For Random Effects ANOVA Analysis Ignoring Clustering

```
TITLE:      Random effects ANOVA data
            Ignoring clustering

DATA:       FILE = anova.dat;

VARIABLE:   NAMES = y cluster;
            USEV = y;
!           CLUSTER = cluster;

ANALYSIS:
```

Output Excerpts Random Effects ANOVA Analysis Ignoring Clustering

Model Results

	Estimates	S.E.	Est./S.E.
Means			
Y	0.003	**0.022**	0.131
Variances			
Y	0.990	0.031	31.623

Note: The estimated mean has SE = 0.022 instead of the correct 0.038

Further Readings On Complex Survey Data

Asparouhov, T. (2005). Sampling weights in latent variable modeling. Structural Equation Modeling, 12, 411-434.

Chambers, R.L. & Skinner, C.J. (2003). Analysis of survey data. Chichester: John Wiley & Sons.

Kaplan, D. & Ferguson, A.J (1999). On the utilization of sample weights in latent variable models. Structural Equation Modeling, 6, 305-321.

Korn, E.L. & Graubard, B.I (1999). Analysis of health surveys. New York: John Wiley & Sons.

Patterson, B.H., Dayton, C.M. & Graubard, B.I. (2002). Latent class analysis of complex sample survey data: application to dietary data. Journal of the American Statistical Association, 97, 721-741.

Skinner, C.J., Holt, D. & Smith, T.M.F. (1989). Analysis of complex surveys. West Sussex, England: Wiley.

Further Readings On Complex Survey Data

Stapleton, L. (2002). The incorporation of sample weights into multilevel structural equation models. Structural Equation Modeling, 9, 475-502.

See also the Mplus Complex Survey Data Project: http://www.statmodel.com/resrchpap.shtml

Two-Level Regression Analysis

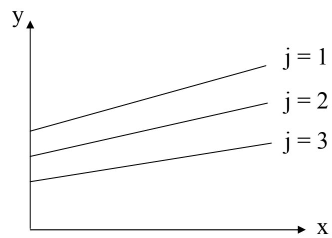
Cluster-Specific Regressions

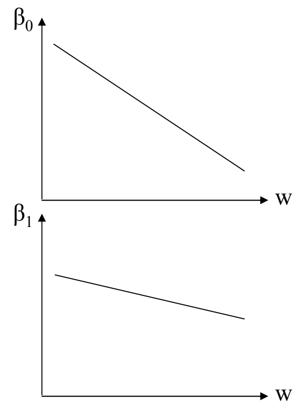
Individual *i* in cluster *j*

(1) $y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}$

(2a) $\beta_{0j} = \gamma_{00} + \gamma_{01}\, w_j + u_{0j}$

(2b) $\beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}$





Two-Level Regression Analysis With Random Intercepts And Random Slopes In Multilevel Terms

Two-level analysis (individual *i* in cluster *j*):

y_{ij} : individual-level outcome variable
x_{ij} : individual-level covariate
w_j : cluster-level covariate

Random intercepts, random slopes:

$$Level\ 1\ (Within)\ :\ y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}, \quad (1)$$

$$Level\ 2\ (Between)\ :\ \beta_{0j} = \gamma_{00} + \gamma_{01}\, w_j + u_{0j}, \quad (2a)$$

$$Level\ 2\ (Between)\ :\ \beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}. \quad (2b)$$

- Mplus gives the same estimates as HLM/MLwiN ML (not REML):
  - $V(r)$ (residual variance for level 1)
  - $\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}, V(u_0), V(u_1), Cov(u_0, u_1)$ (level 2)

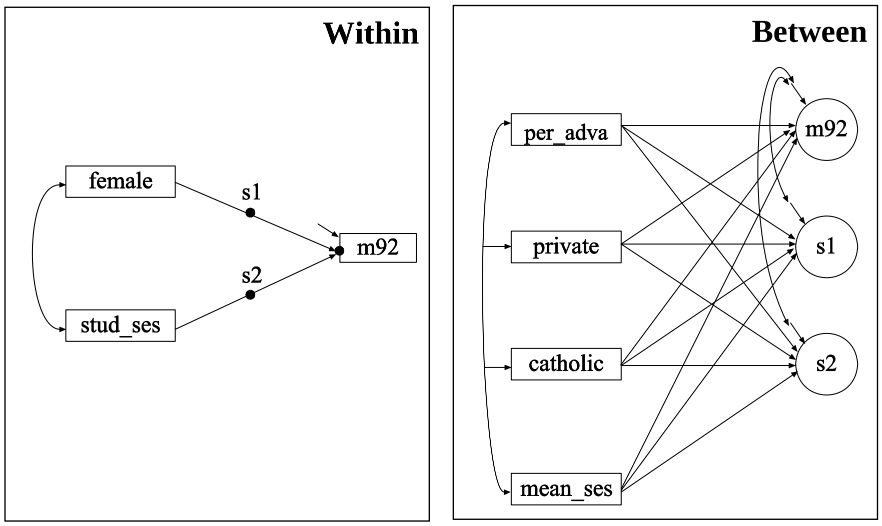
WITHIN And BETWEEN Options Of The VARIABLE Command

- WITHIN
  - Measured on individual level
  - Modeled on within
  - No variance on between
- BETWEEN
  - Measured on cluster level
  - Modeled on between
- Not on WITHIN or BETWEEN
  - Measured on individual level
  - Modeled on within and between

NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school, n = 14,217
  - Variables—reading, math, science, history-citizenship-geography, and background variables

NELS Math Achievement Regression



Input For NELS Math Achievement Regression

```
TITLE:     NELS math achievement regression

DATA:      FILE IS completev2.dat;
           ! National Education Longitudinal Study (NELS)
           FORMAT IS f8.0 12f5.2 f6.3 f11.4 23f8.2
           f18.2 f8.0 4f8.2;

VARIABLE:  NAMES ARE school r88 m88 s88 h88 r90 m90 s90 h90 r92
           m92 s92 h92 stud_ses f2pnlwt transfer minor coll_asp
           algebra retain aca_back female per_mino hw_time
           salary dis_fair clas_dis mean_col per_high unsafe
           num_frie teaqual par_invo ac_track urban size rural
           private mean_ses catholic stu_teac per_adva tea_exce
           tea_res;

           USEV = m92 female stud_ses per_adva private catholic
           mean_ses;

           !per_adva = percent teachers with an MA or higher

           WITHIN = female stud_ses;
           BETWEEN = per_adva private catholic mean_ses;
           MISSING = blank;
           CLUSTER = school;
           CENTERING = GRANDMEAN (stud_ses per_adva mean_ses);
```

Input For NELS Math Achievement Regression (Continued)

```
ANALYSIS: TYPE = TWOLEVEL RANDOM;

MODEL:
          %WITHIN%
          s1 | m92 ON female;
          s2 | m92 ON stud_ses;

          %BETWEEN%
          m92 s1 s2 ON per_adva private catholic mean_ses;
          m92 WITH s1 s2;

OUTPUT:   TECH8 SAMPSTAT;
```


Output Excerpts NELS Math Achievement Regression

N = 10,933

Summary of Data

```
Number of clusters     902

Size (s)  Cluster ID with Size s

 1        89863  75862  52654   1995  32661  89239  56214
 2        41743  81263  45025  26790  60281  82860  56241  21474
           4570  27159  11662  87842  38454
 3        65407  61407  83048  42640  41412  67708  83085  39685
          40402  93469  98582  68595  11517  17543  75498  81069
          66512
 4        31646  68153  85508  26234  83390  60835  74400  20770
           5095  10904  93569  38063  86733  66125  51670  10910
          98461  44395  95317  64112  50880  77381  12835  47555
           9208  93859  35719  67574  20048  34139  25784  80675
 5        14464  74791  18219  10468  72193  97616  15773    877
           9471  83234  68254  68028  70718   3496   6842  45854
```

Output Excerpts NELS Math Achievement Regression (Continued)

```
    22        79570   15426   97947   93599   85125   10926    4603
    23         6411   60328   70024   67835
    24        36988   22874   50626   19091
    25        56619   59710   34292   18826   62209
    26        44586   67832   16515
    27        82887
    28          847   76909
    30        36177
    31        12786   53660   47120   94802
    32        80553
    34        53272
    36        89842   31572
    42        99516
    43        75115

Average cluster size  12.187
Estimated Intraclass Correlations for the Y Variables
          Intraclass
Variable Correlation
M92         0.107
```


Output Excerpts NELS Math Achievement Regression (Continued)

Tests of Model Fit

```
Loglikelihood
          H0 Value                    -39390.404
Information Criteria
          Number of Free parameters          21
          Akaike (AIC)                 78822.808
          Bayesian (BIC)               78976.213
          Sample-Size Adjusted BIC     78909.478
             (n* = (n + 2) / 24)
```

Model Results

```
                   Estimates     S.E.    Est./S.E.
Within Level
Residual
Variances
   M92                70.577      1.149     61.442
Between Level
S1          ON
   PER_ADVA            0.084      0.841      0.100
   PRIVATE            -0.134      0.844     -0.159
   CATHOLIC           -0.736      0.780     -0.944
   MEAN_SES           -0.232      0.428     -0.542
```

Output Excerpts NELS Math Achievement Regression (Continued)

```
S2          ON          Estimates      S.E.   Est./S.E.
   PER_ADVA                 1.348     0.521      2.587
   PRIVATE                 -1.890     0.706     -2.677
   CATHOLIC                -1.467     0.562     -2.612
   MEAN_SES                 1.031     0.283      3.640
M92         ON
   PER_ADVA                 0.195     0.727      0.268
   PRIVATE                  1.505     1.108      1.358
   CATHOLIC                 0.765     0.650      1.178
   MEAN_SES                 3.912     0.399      9.814
S1           WITH
   M92                     -4.456     1.007     -4.427
 S2          WITH
   M92                      0.128     0.399      0.322
Intercepts
   M92                     55.136     0.185    297.248
   S1                      -0.819     0.211     -3.876
   S2                       4.841     0.152     31.900
 Residual Variances
   M92                      8.679     1.003      8.649
   S1                       5.740     1.411      4.066
   S2                       0.307     0.527      0.583
```


Cross-Level Influence

Between-level (level 2) variable w influencing within-level (level 1) y variable:

Random intercept

$$y_{ij} = \beta_{0j} + \beta_1 x_{ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01} w_j + u_{0j}$$

Mplus:

```
MODEL:
        %WITHIN%;
        y ON x;    ! estimates beta1
        %BETWEEN%;
        y ON w;    ! y is the same as beta0j
                   ! estimates gamma01
```

Cross-Level Influence (Continued)

Cross-level interaction, or between-level (level 2) variable moderating a within level (level 1) relationship:

Random slope

$$y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}$$

Mplus:

```
MODEL:

        %WITHIN%;
        beta1 | y ON x;
        %BETWEEN%;
        beta1 ON w;      ! estimates gamma11
```


Random Slopes: Varying Variances

$$y_{ij} = \beta_{0j} + \beta_{1j}\, x_{ij} + r_{ij}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\, w_j + u_{1j}$$

$$V(y_{ij} \mid x_{ij}, w_j) = V(u_{1j})\, x_{ij}^2 + V(r_{ij})$$

The variance varies as a function of the x_{ij} values.

So there is no single population covariance matrix for testing the model fit

Random Slopes In Mplus

Mplus allows random slopes for predictors that are

- Observed covariates
- Observed dependent variables
- Continuous latent variables

Two-Level Variable Decomposition

$$y_{ij} = \beta_{0j} + \beta_1 x_{ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}\bar{x}_{\cdot j} + u_{0j}$$

A random intercept model is the same as decomposing y_{ij} into two uncorrelated components

$$y_{ij} = y_{wij} + y_{bj}$$

where

$$y_{wij} = \beta_1 x_{ij} + r_{ij}$$

$$y_{bj} = \beta_{0j} = \gamma_{00} + \gamma_{01}\bar{x}_{\cdot j} + u_{0j}$$

Two-Level Variable Decomposition (Continued)

The same decomposition can be made for x_{ij},

$$x_{ij} = x_{wij} + x_{bj}$$

where x_{wij} and x_{bj} are latent covariates,

$$y_{wij} = \beta_w x_{wij} + r_{ij}$$

$$y_{bj} = \gamma_{00} + \beta_b x_{bj} + u_{0j}$$

Mplus can work with either manifest or latent covariates.

See also User's Guide example 9.1.b

Bias With Manifest Covariates

Comparing the manifest and latent covariate approach shows a bias in the manifest between-level slope

$$E(\hat{\gamma}_{01}) - \beta_b = (\beta_w - \beta_b)\frac{1}{s}\frac{(1 - icc_x)}{icc_x + (1 - icc_x)/s}$$

Bias increases with decreasing cluster size s and decreasing icc_x.

Example: $(\beta_w - \beta_b) = 0.5$, $s = 10$, $icc_x = 0.1$
gives bias = 0.25

No bias for latent covariate approach
Asparouhov-Muthen (2006), Ludtke et al. (2008)

Further Readings On Multilevel Regression Analysis

Enders, C.K. & Tofighi, D. (2007). Centering predictor variables in cross-sectional multilevel models: A new look at an old Issue. Psychological Methods, 12, 121-138.

Lüdtke, O., Marsh, H.W., Robitzsch, A., Trautwein, U., Asparouhov, T., & Muthén, B. (2008). The multilevel latent covariate model: A new, more reliable approach to group-level effects in contextual studies. Psychological Methods, 13, 203-229.

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

Logistic And Probit Regression

Categorical Outcomes: Logit And Probit Regression

Probability varies as a function of x variables (here x_1, x_2)

$$P(u = 1 \mid x_1, x_2) = F[\beta_0 + \beta_1 x_1 + \beta_2 x_2], \qquad (22)$$

$P(u = 0 \mid x_1 , x_2) = 1 - P[u = 1 \mid x_1 , x_2]$, where F[z] is either the standard normal ($\Phi[z]$) or logistic ($1/[1 + e^{-z}]$) distribution function.

Example: Lung cancer and smoking among coal miners

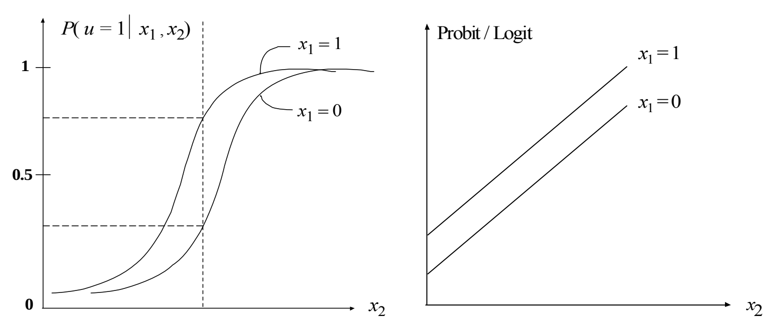
u lung cancer ($u = 1$) or not ($u = 0$)

x_1 smoker ($x_1 = 1$), non-smoker ($x_1 = 0$)

x_2 years spent in coal mine

Categorical Outcomes: Logit And Probit Regression

$$P(u = 1 \mid x_1, x_2) = F [\beta_0 + \beta_1 x_1 + \beta_2 x_2], \qquad (22)$$



Interpreting Logit And Probit Coefficients

- Sign and significance
- Odds and odds ratios
- Probabilities

Logistic Regression And Log Odds

Odds $(u = 1 \mid x) = P(u = 1 \mid x) \,/\, P(u = 0 \mid x)$
$= P(u = 1 \mid x) \,/\, (1 - P(u = 1 \mid x))$.

The logistic function

$$P(u = 1 \mid x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

gives a log odds linear in x,

$$logit = log\,[odds\,(u = 1 \mid x)] = log\,[P(u = 1 \mid x) \,/\, (1 - P(u = 1 \mid x))]$$

$$= \log\left[\frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \,/\, (1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}})\right]$$

$$= \log\left[\frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} * \frac{1 + e^{-(\beta_0 + \beta_1 x)}}{e^{-(\beta_0 + \beta_1 x)}}\right]$$

$$= \log\left[e^{(\beta_0 + \beta_1 x)}\right] = \beta_0 + \beta_1 x$$

Logistic Regression And Log Odds (Continued)

- *logit* = *log odds* = $\beta_0 + \beta_1 x$
- When *x* changes one unit, the *logit* (*log odds*) changes β_1 units
- When *x* changes one unit, the *odds* changes e^{β_1} units

Two-Level Logistic Regression

Two-Level Logistic Regression Model

With *i* denoting individual and *j* denoting cluster,

$$P(u_{ij} = 1 \mid x_{ij}) = \frac{1}{1 + e^{-(\beta_{0j} + \beta_{1j} x_{ij})}}$$

$$logit_{ij} = log\left[\frac{P(u_{ij} = 1 \mid x)}{P(u_{ij} = 0 \mid x)}\right] = \beta_{0j} + \beta_{1j}\, x_{ij}$$

where

$$\beta_{0j} = \beta_0 + u_{0j}$$
$$\beta_{1j} = \beta_1 + u_{1j}$$

High/low β_{0j} value means high/low logit (high/low log odds)

Predicting Juvenile Delinquency From First Grade Aggressive Behavior

- Cohort 1 data from the Johns Hopkins University Preventive Intervention Research Center
- n= 1,084 students in 40 classrooms, Fall first grade
- Covariates: gender and teacher-rated aggressive behavior

Input For Two-Level Logistic Regression

```
TITLE:
        Hopkins Cohort 1 2-level logistic regression
DATA:
        FILE = Cohort1_classroom_ALL.DAT;
VARIABLE:
        NAMES =  prcid juv99 gender stub1F bkRule1F harmO1F
                 bkThin1F yell1F takeP1F fight1F lies1F
                 tease1F;
        ! juv99:  juvenile delinquency record by age 18
        CLUSTER = classrm;
        USEVAR = juv99 male aggress;
        CATEGORICAL =  juv99;
        MISSING = ALL (999);
        WITHIN = male aggress;
DEFINE:
        male = 2 - gender;
        aggress = stub1F + bkRule1F + harmO1F + bkThin1F +
                 yell1F + takeP1F + fight1F + lies1F + tease1F;
```


Input For Two-Level Logistic Regression (Continued)

```
ANALYSIS:
        TYPE = TWOLEVEL;
        PROCESS = 2;
MODEL:
        %WITHIN%
        juv99 ON male aggress;
        %BETWEEN%
OUTPUT:
        TECH1 TECH8;
```

Output Excerpts Two-Level Logistic Regression

```
MODEL RESULTS
                     Estimates    S.E   Est./S.E.
Within Level
  JUV99         ON
    MALE                1.071     0.149     7.193
    AGGRESS             0.060     0.010     6.191

Between Level
Thresholds
  JUV99$1               2.981     0.205    14.562

Variances
  JUV99                 0.807     0.250     3.228
```


Understanding The Between-Level Intercept Variance

- Intra-class correlation
  - ICC = 0.807/(π^2/3+ 0.807) = 0.20
- Odds ratios
  - Larsen & Merlo (2005). Appropriate assessment of neighborhood effects on individual health: Integrating random and fixed effects in multilevel logistic regression. American Journal of Epidemiology, 161, 81-88.
  - Larsen proposes MOR:
    "Consider two persons with the same covariates, chosen randomly from two different clusters. The MOR is the median odds ratio between the person of higher propensity and the person of lower propensity."

    MOR = exp($\sqrt{(2^* \sigma^2)}$ * Φ^{-1} (0.75))

    In the current example, ICC = 0.20, MOR = 2.36
- Probabilities
  - Compare α_j=1 SD and α_k=-1 SD from the mean
  - For males at the aggression mean the probability varies from 0.14 to 0.50

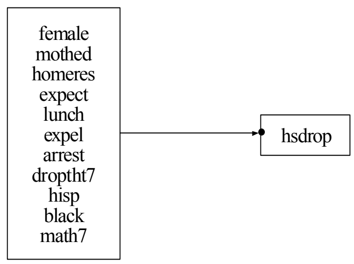
Two-Level Path Analysis

LSAY Data

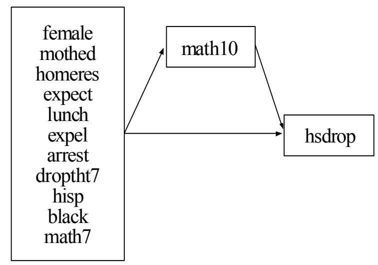
- Longitudinal Study of American Youth
- Math and science testing in grades 7 – 12
- Interest in high school dropout
- Data for 2,213 students in 44 public schools

A Path Model With A Binary Outcome And A Mediator With Missing Data

Logistic Regression

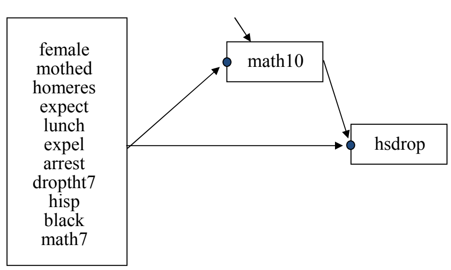


Path Model

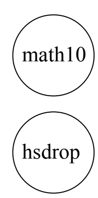


Two-Level Path Analysis

Within



Between



Input For A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable

```
TITLE:      a twolevel path analysis with a categorical outcome
            and missing data on the mediating variable
DATA:       FILE = lsayfull_dropout.dat;
VARIABLE:   NAMES = female mothed homeres math7 math10 expel
            arrest hisp black hsdrop expect lunch droptht7
            schcode;
            CATEGORICAL = hsdrop;
            CLUSTER = schcode;
            WITHIN = female mothed homeres expect math7 lunch
            expel arrest droptht7 hisp black;
ANALYSIS:   TYPE = TWOLEVEL;
            ESTIMATOR = ML;
            ALGORITHM = INTEGRATION;
            INTEGRATION = MONTECARLO (500);
```


Input For A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

```
MODEL:
            %WITHIN%
            hsdrop ON female mothed homeres expect math7 math10
            lunch expel arrest droptht7 hisp black;
            math10 ON female mothed homeres expect math7 lunch
            expel arrest droptht7 hisp black;

            %BETWEEN%
            hsdrop*1; math10*1;

OUTPUT:     PATTERNS SAMPSTAT STANDARDIZED TECH1 TECH8;
```

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable

Summary Of Data

```
          Number of patterns            2
          Number of clusters           44

     Size (s)        Cluster ID with Size s
        12              304
        13              305
        36              307     122
        38              106     112
        39              138     109
        40              103
        41              308
        42              146     120
        43              102     101
        44              303     143
        45              141
```


Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

```
     Size (s)        Cluster ID with Size s
        46              144
        47              140
        49              108
        50              126     111     110
        51              127     124
        52              137     117     147     118     301     136
        53              142     131
        55              145     123
        57              135     105
        58              121
        59              119
        73              104
        89              302
        93              309
       118              115
```

Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
Within Level					
HSDROP ON					
FEMALE	0.323	0.171	1.887	0.323	0.077
MOTHED	-0.253	0.103	-2.457	-0.253	-0.121
HOMERES	-0.077	0.055	-1.401	-0.077	-0.061
EXPECT	-0.244	0.065	-3.756	-0.244	-0.159
MATH7	-0.011	0.015	-0.754	-0.011	-0.055
MATH10	-0.031	0.011	-2.706	-0.031	-0.197
LUNCH	0.008	0.006	1.324	0.008	0.074
EXPEL	0.947	0.225	4.201	0.947	0.121
ARREST	0.068	0.321	0.212	0.068	0.007
DROPTHT7	0.757	0.284	2.665	0.757	0.074
HISP	-0.118	0.274	-0.431	-0.118	-0.016
BLACK	-0.086	0.253	-0.340	-0.086	-0.013

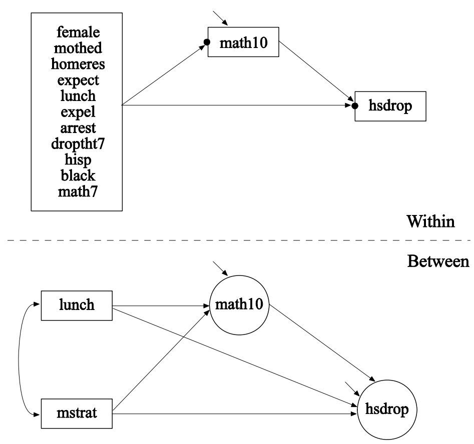
Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
MATH10 ON					
FEMALE	-0.841	0.398	-2.110	-0.841	-0.031
MOTHED	0.263	0.215	1.222	0.263	0.020
HOMERES	0.568	0.136	4.169	0.568	0.070
EXPECT	0.985	0.162	6.091	0.985	0.100
MATH7	0.940	0.023	40.123	0.940	0.697
LUNCH	-0.039	0.017	-2.308	-0.039	-0.059
EXPEL	-1.293	0.825	-1.567	-1.293	-0.026
ARREST	-3.426	1.022	-3.353	-3.426	-0.054
DROPTHT7	-1.424	1.049	-1.358	-1.424	-0.022
HISP	-0.501	0.728	-0.689	-0.501	-0.010
BLACK	-0.369	0.733	-0.503	-0.369	-0.009

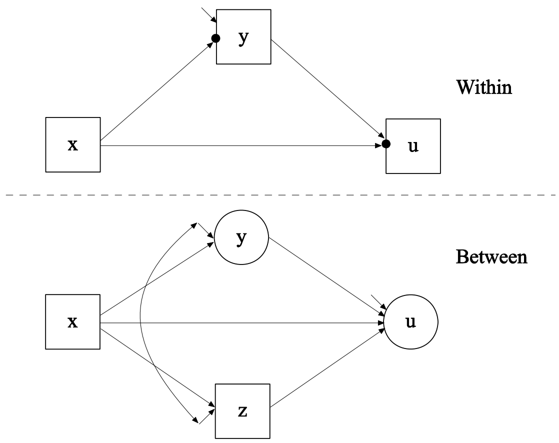
Output Excerpts A Two-Level Path Analysis Model With A Categorical Outcome And Missing Data On The Mediating Variable (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Residual Variances					
MATH10	62.010	2.162	28.683	62.010	0.341
Between Level					
Means					
MATH10	10.226	1.340	7.632	10.226	5.276
Thresholds					
HSDROP$1	-1.076	0.560	-1.920		
Variances					
HSDROP	0.286	0.133	2.150	0.286	1.000
MATH10	3.757	1.248	3.011	3.757	1.000

Two-Level Path Analysis Model Variation

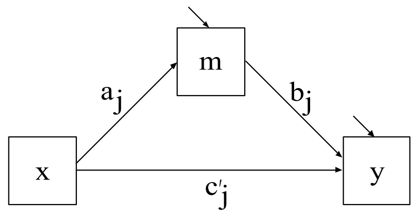


Model Diagram For Path Analysis With Between-Level Dependent Variable



Two-Level Mediation With Random Slopes

Two-Level Mediation



Indirect effect:

$$\alpha * \beta + Cov\,(a_j, b_j)$$

Bauer, Preacher & Gil (2006). Conceptualizing and testing random indirect effects and moderated mediation in multilevel models: New procedures and recommendations. Psychological Methods, 11, 142-163.

Input For Two-Level Mediation

```
MONTECARLO:
        NAMES ARE y m x;
        WITHIN = x;
        NOBSERVATIONS = 1000;
        NCSIZES = 1;
        CSIZES = 100 (10);
        NREP = 100;

MODEL POPULATION:
        %WITHIN%
        c | y ON x;
        b | y ON m;
        a | m ON x;
        x*1; m*1; y*1;
        %BETWEEN%
        y WITH m*0.1 b*0.1 a*0.1 c*0.1;
        m WITH b*0.1 a*0.1 c*0.1;
        a WITH b*0.1 c*0.1;
        b WITH c*0.1;
        y*1 m*1 a*1 b*1 c*1;
        [a*0.4 b*0.5 c*0.6];
```

Input For Two-Level Mediation (Continued)

```
ANALYSIS:
       TYPE = TWOLEVEL RANDOM;
MODEL:
        %WITHIN%
        c | y ON x;
        b | y ON m;
        a | m ON x;
        m*1; y*1;
        %BETWEEN%
        y WITH M*0.1 b*0.1 a*0.1 c*0.1;
        m WITH b*0.1 a*0.1 c*0.1;
        a WITH b*0.1 (cab);
        a WITH c*0.1;
        b WITH c*0.1;
        y*1 m*1 a*1 b*1 c*1;
        [a*0.4] (ma);
        [b*0.5] (mb);
        [c*0.6];

 MODEL CONSTRAINT:
        NEW(m*0.3);
        m=ma*mb+cab;
```


Output Excerpts Two Level Mediation

	Estimates			S.E.	M. S. E.	95%	% Sig
	Population	Average	Std.Dev.	Average		Cover	Coeff
Within Level							
Residual variances							
Y	1.000	1.0020	0.0530	0.0530	0.0028	0.960	1.000
M	1.000	1.0011	0.0538	0.0496	0.0029	0.910	1.000
Between Level							
Y WITH							
B	0.100	0.1212	0.1246	0.114	0.0158	0.910	0.210
A	0.100	0.1086	0.1318	0.1162	0.0173	0.910	0.190
C	0.100	0.0868	0.1121	0.1237	0.0126	0.940	0.090
M WITH							
B	0.100	0.1033	0.1029	0.1085	0.0105	0.940	0.120
A	0.100	0.0815	0.1081	0.1116	0.0119	0.950	0.070
C	0.100	0.1138	0.1147	0.1165	0.0132	0.970	0.160
A WITH							
B	0.100	0.0964	0.1174	0.1101	0.0137	0.920	0.150
C	0.100	0.0756	0.1376	0.1312	0.0193	0.910	0.110

Output Excerpts Two-Level Mediation (Continued)

B WITH							
C	0.100	0.0892	0.1056	0.1156	0.0112	0.960	0.070
Y WITH							
M	0.100	0.1034	0.1342	0.1285	0.0178	0.940	0.140
Means							
Y	0.000	0.0070	0.1151	0.1113	0.0132	0.950	0.050
M	0.000	-0.0031	0.1102	0.1056	0.0120	0.950	0.050
C	0.600	0.5979	0.1229	0.1125	0.0150	0.930	1.000
B	0.500	0.5022	0.1279	0.1061	0.0162	0.890	1.000
A	0.400	0.3854	0.0972	0.1072	0.0096	0.970	0.970
Variances							
Y	1.000	1.0071	0.1681	0.1689	0.0280	0.910	1.000
M	1.000	1.0113	0.1782	0.1571	0.0316	0.930	1.000
C	1.000	0.9802	0.1413	0.1718	0.0201	0.980	1.000
B	1.000	0.9768	0.1443	0.1545	0.0212	0.950	1.000
A	1.000	1.0188	0.1541	0.1587	0.0239	0.950	1.000
New/Additional Parameters							
M	0.300	0.2904	0.1422	0.1316	0.0201	0.950	0.550

Two-Level Factor Analysis

Two-Level Factor Analysis

- Recall random effects ANOVA (individual *i* in cluster *j*):

$$y_{ij} = \nu + \eta_j + \varepsilon_{ij} = y_{B_j} + y_{W_{ij}}$$

- Two-level factor analysis ($r = 1, 2, \ldots, p$ items):

$$y_{rij} = \nu_r + \lambda_{B_r} \eta_{B_j} + \varepsilon_{B_{rj}} + \lambda_{W_r} \eta_{Wij} + \varepsilon_{Wrij}$$

(between-cluster variation) (within-cluster variation)

Two-Level Factor Analysis (Continued)

- Covariance structure:

$$V(\boldsymbol{y}) = V(\boldsymbol{y}_B) + V(\boldsymbol{y}_w) = \boldsymbol{\Sigma}_B + \boldsymbol{\Sigma}_w,$$

$$\boldsymbol{\Sigma}_B = \boldsymbol{\Lambda}_B \; \boldsymbol{\Psi}_B \; \boldsymbol{\Lambda}_B' + \boldsymbol{\Theta}_B,$$

$$\boldsymbol{\Sigma}_W = \boldsymbol{\Lambda}_W \; \boldsymbol{\Psi}_W \; \boldsymbol{\Lambda}_W' + \boldsymbol{\Theta}_W .$$

- Two interpretations:
  - variance decomposition, including decomposing the residual
  - random intercept model

Two-Level Factor Analysis And Design Effects

Muthén & Satorra (1995; Sociological Methodology): Monte Carlo study using two-level data (200 clusters of varying size and varying intraclass correlations), a latent variable model with 10 variables, 2 factors, conventional ML using the regular sample covariance matrix S_T, and 1,000 replications (d.f. = 34).

$$\Lambda_B = \Lambda_W = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad \Psi_B, \Theta_B \text{ reflecting different icc's}$$

$$y_{ij} = \nu + \Lambda(\eta_{B_j} + \eta_{W_{ij}}) + \varepsilon_{B_j} + \varepsilon_{W_{ij}}$$

$$V(y) = \Sigma_B + \Sigma_W = \Lambda(\Psi_B + \Psi_W)\,\Lambda' + \Theta_B + \Theta_W$$

Two-Level Factor Analysis And Design Effects (Continued)

Inflation of χ^2 due to clustering

Intraclass Correlation		Cluster Size 7	15	30	60
0.05					
	Chi-square mean	35	36	38	41
	Chi-square var	68	72	80	96
	5%	5.6	7.6	10.6	20.4
	1%	1.4	1.6	2.8	7.7
0.10					
	Chi-square mean	36	40	46	58
	Chi-square var	75	89	117	189
	5%	8.5	16.0	37.6	73.6
	1%	1.0	5.2	17.6	52.1
0.20					
	Chi-square mean	42	52	73	114
	Chi-square var	100	152	302	734
	5%	23.5	57.7	93.1	99.9
	1%	8.6	35.0	83.1	99.4

Two-Level Factor Analysis And Design Effects (Continued)

- Regular analysis, ignoring clustering
  - Inflated chi-square, underestimated SE's
- TYPE = COMPLEX
  - Correct chi-square and SE's but only if model aggregates, e.g. $\Lambda_B = \Lambda_W$
- TYPE = TWOLEVEL
  - Correct chi-square and SE's

SIMS Variance Decomposition

The Second International Mathematics Study (SIMS; Muthén, 1991, JEM).

- National probability sample of school districts selected proportional to size; a probability sample of schools selected proportional to size within school district, and two classes randomly drawn within each school
- 3,724 students observed in 197 classes from 113 schools with class sizes varying from 2 to 38; typical class size of around 20
- Eight variables corresponding to various areas of eighth-grade mathematics
- Same set of items administered as a pretest in the Fall of eighth grade and as a posttest in the Spring.

SIMS Variance Decomposition (Continued)

Muthén (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement, 28, 338-354.

- Research questions: "The substantive questions of interest in this article are the variance decomposition of the subscores with respect to within-class student variation and between-class variation and the change of this decomposition from pretest to posttest. In the SIMS … such variance decomposition relates to the effects of tracking and differential curricula in eighth-grade math. On the one hand, one may hypothesize that effects of selection and instruction tend to increase between-class variation relative to within-class variation, assuming that the classes are homogeneous, have different performance levels to begin with, and show faster growth for higher initial performance level. On the other hand, one may hypothesize that eighth-grade exposure to new topics will increase individual differences among students within each class so that posttest within-class variation will be sizable relative to posttest between-class variation."

SIMS Variance Decomposition (Continued)

$$y_{rij} = \nu_r + \lambda_{Br}\,\eta_{Bj} + \varepsilon_{Brj} + \lambda_{wr}\,\eta_{wij} + \varepsilon_{wrij}$$

$$V(y_{rij}) = \quad BF \;+ BE \;+ WF \;+ WE$$

Between reliability: BF / (BF + BE)
– BE often small (can be fixed at 0)

Within reliability: WF / (WF + WE)
– sum of a small number of items gives a large WE

Intraclass correlation:
ICC = (BF + BE) / (BF + BE + WF + WE)

Large measurement error → large WE → small ICC

True ICC = BF / (BF + WF)

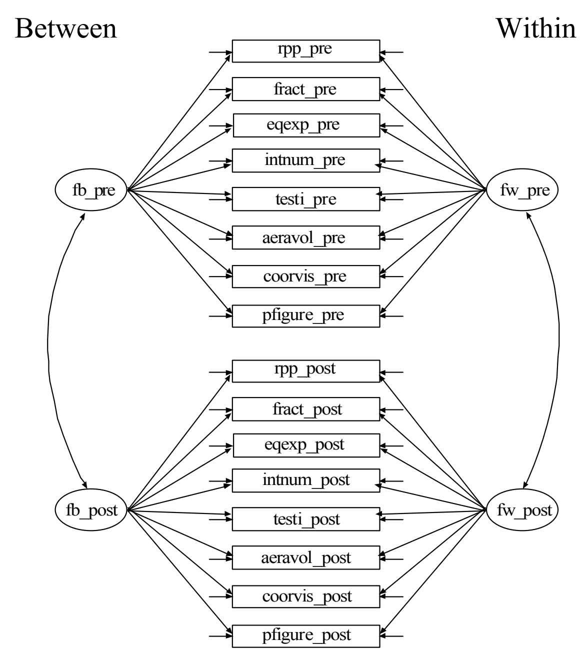


Table 4: Variance Decomposition of SIMS Achievement Scores (percentages of total variance in parenthesis)

		ANOVA								FACTOR ANALYSIS			
		Pretest			Posttest			% Increase In Variance		Error-free Prop. Between		Error-free % Increase In Variance	
	Number of Items	Between	Within	Prop-Between	Between	Within	Prop-Between	Between	Within	Pre	Post	Between	Within
RPP	8	1.542 (34.0)	2.990 (66.0)	.34	2.084 (38.5)	3.326 (61.5)	.38	35	11	.54	.52	29	41
FRACT	8	1.460 (38.2)	2.366 (61.8)	.38	1.906 (40.8)	2.767 (59.2)	.41	31	17	.60	.58	29	41
EQEXP	6	.543 (26.9)	1.473 (73.1)	.27	1.041 (38.7)	1.646 (61.3)	.39	92	18	.65	.64	113	117
INTNUM	2	.127 (25.2)	.358 (70.9)	.29	.195 (30.6)	.442 (69.4)	.31	54	24	.63	.61	29	41
TESTI	5	.580 (33.3)	1.163 (66.7)	.33	.664 (34.5)	1.258 (65.5)	.34	15	8	.58	.56	29	41
AREAVOL	2	.094 (17.2)	.451 (82.8)	.17	.156 (24.1)	.490 (75.9)	.24	66	9	.54	.52	29	41
COORVIS	3	.173 (20.9)	.656 (79.1)	.21	.275 (28.7)	.680 (68.3)	.32	59	4	.57	.55	29	41
PFIGURE	5	.363 (22.9)	1.224 (77.1)	.23	.711 (42.9)	1.451 (67.1)	.33	96	19	.60	.54	87	136

Exploratory Factor Analysis Of Aggression Items

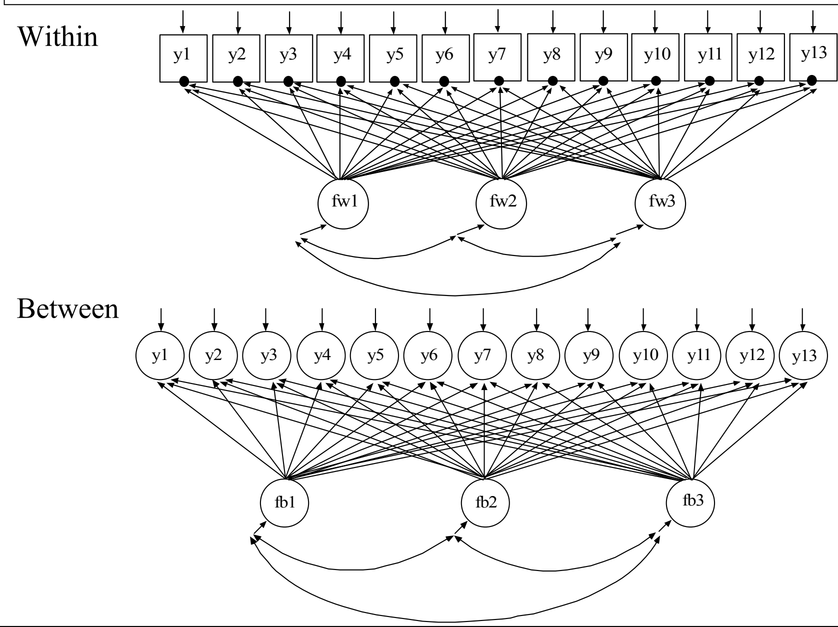
Item Distributions for Cohort 3: Fall 1st Grade (n=362 males in 27 classrooms)

	Almost Never (scored as 1)	*Rarely (scored as 2)*	*Sometimes (scored as 3)*	*Often (scored as 4)*	*Very Often (scored as 5)*	*Almost Always (scored as 6)*
Stubborn	42.5	21.3	18.5	7.2	6.4	4.1
Breaks Rules	37.6	16.0	22.7	7.5	8.3	8.0
Harms Others	69.3	12.4	9.40	3.9	2.5	2.5
Breaks Things	79.8	6.60	5.20	3.9	3.6	0.8
Yells at Others	61.9	14.1	11.9	5.8	4.1	2.2
Takes Others' Property	72.9	9.70	10.8	2.5	2.2	1.9
Fights	60.5	13.8	13.5	5.5	3.0	3.6
Harms Property	74.9	9.90	9.10	2.8	2.8	0.6
Lies	72.4	12.4	8.00	2.8	3.3	1.1
Talks Back to Adults	79.6	9.70	7.80	1.4	0.8	1.4
Teases Classmates	55.0	14.4	17.7	7.2	4.4	1.4
Fights With Classmates	67.4	12.4	10.2	5.0	3.3	1.7
Loses Temper	61.6	15.5	13.8	4.7	3.0	1.4

Hypothesized Aggressiveness Factors

- Verbal aggression
  - Yells at others
  - Talks back to adults
  - Loses temper
  - Stubborn
- Property aggression
  - Breaks things
  - Harms property
  - Takes others' property
  - Harms others
- Person aggression
  - Fights
  - Fights with classmates
  - Teases classmates

Two-Level Factor Analysis



Reasons For Finding Dimensions

Different dimensions may have different

- Predictors
- Effects on later events
- Growth curves
- Treatment effects

Categorical Outcomes, Latent Dimensions, And Computational Demand

- ML requires numerical integration (see end of Topic 8)
  - increasingly time consuming for increasing number of continuous latent variables and increasing sample size
- Bayes analysis
- Limited information weighted least squares estimation

Two-Level Weighted Least Squares

- New simple alternative (Asparouhov & Muthén, 2007):
  - computational demand virtually independent of number of factors/random effects
  - high-dimensional integration replaced by multiple instances of one- and two-dimensional integration
  - possible to explore many different models in a time-efficient manner
  - generalization of the Muthen (1984) single-level WLS
  - variables can be categorical, continuous, censored, combinations
  - residuals can be correlated (no conditional independence assumption)
  - model fit chi-square testing
  - can produce unrestricted level 1 and level 2 correlation matrices for EFA

Input For Two-Level EFA of Aggression Using WLSM And Geomin Rotation

```
TITLE:    two-level EFA of 13 TOCA aggression items

DATA:     FILE IS Muthen.dat;

VARIABLE: NAMES ARE id race lunch312 gender u1-u13 sgsf93;
          MISSING are all (999);
          USEOBS = gender eq 1;  !males
          USEVARIABLES = u1-u13;
          CATEGORICAL = u1-u13;
          CLUSTER = sgsf93;

ANALYSIS: TYPE = TWOLEVEL EFA 1 3 UW 1 3 UB;
          PROCESS = 4;

SAVEDATA: SWMATRIX = sw.dat;
```


Output Excerpts Two-Level EFA of Aggression Using WLSM And Geomin Rotation

```
Number of clusters                            27

Average cluster size      13.407

Estimated Intraclass Correlations for the Y Variables
```

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
U1	0.110	U2	0.121	U3	0.208
U4	0.378	U5	0.213	U6	0.250
U7	0.161	U8	0.315	U9	0.208
U10	0.140	U11	0.178	U12	0.162
U13	0.172				

Two-Level EFA Model Test Result For Aggressive-Disruptive Items

Within-level Factors	Between-level Factors	Df	Chi-Square	CFI	RMSEA
unrestricted	1	65	66(p=0.43)	1.000	0.007
1	1	130	670	0.991	0.107
2	1	118	430	0.995	0.084
3	1	107	258	0.997	0.062
4*	1	97	193	0.998	0.052

*4th factor has no significant loadings

Two-Level EFA Of Aggressive-Disruptive Items: Geomin Rotated Factor Loading Matrix

	Within-Level Loadings			Between-Level Loadings
	Property	**Verbal**	**Person**	**General**
Stubborn	0.00	**0.78***	0.01	**0.65***
Breaks Rules	0.31*	0.25*	0.32*	**0.61***
Harms Others and Property	**0.64***	0.12	0.25*	**0.68***
Breaks Things	**0.98***	0.08	-0.12*	**0.98***
Yells At Others	0.11	**0.67***	0.10	**0.93***
Takes Others' Property	**0.73***	-0.15*	0.31*	**0.80***
Fights	0.10	0.03	**0.86***	**0.79***
Harms Property	**0.81***	0.12	0.05	**0.86***
Lies	**0.60***	0.25*	0.10	**0.86***
Talks Back To Adults	0.09	**0.78***	0.05	**0.81***
Teases Classmates	0.12	0.16*	**0.59***	**0.83***
Fights With Classmates	-0.02	0.13	**0.88***	**0.84***
Loses Temper	-0.02	**0.85***	0.05	**0.87***

IRT

Single-level IRT:

$P(u_{ik} = 1 \mid \theta_i, a_k, b_k) = \Phi(a_k\theta_i - b_k)$, (1)

for individual i and item k.

- a is discrimination (slope)
- b is difficulty
- θ is the ability (continuous latent variable)

Two-Level IRT (Fox, 2005)

Two-level IRT (Fox, 2005, p.21; Fox & Glas, 2001):

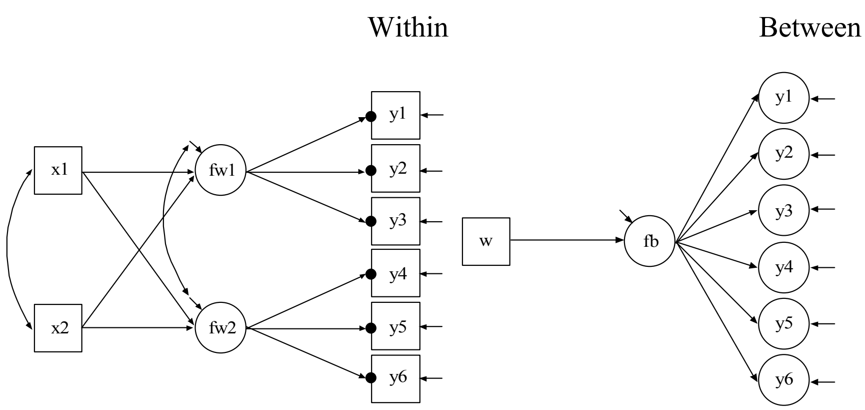
$P(u_{ijk} = 1 \mid \theta_{ij}, a_k, b_k) = \Phi(a_k\theta_{ij} - b_k)$, (1)

for individual i, cluster j, and item k.

$$\left.\begin{array}{l} \theta_{ij} = \beta_{0j} + \beta_{1j}\,SES_{ij} + \beta_{2j}\,Gender_{ij} + \beta_{3j}\,IQ_{ij} + e_{ij}, \\ \beta_{0j} = \gamma_{00} + \gamma_{1j}\,Leader_j + \gamma_{02}\,Climate_j + u_{0j}, \\ \beta_{1j} = \gamma_{10}\,, \\ \beta_{2j} = \gamma_{20}\,, \\ \beta_{3j} = \gamma_{30} \end{array}\right\} \quad (21)$$

Two-Level Factor Analysis With Covariates

Two-Level Factor Analysis With Covariates



Input For Two-Level Factor Analysis With Covariates

```
TITLE:     this is an example of a two-level CFA with
           continuous factor indicators with two factors on the
           within level and one factor on the between level
DATA:      FILE IS ex9.8.dat;
VARIABLE:  NAMES ARE y1-y6 x1 x2 w clus;
           WITHIN = x1 x2;
           BETWEEN = w;
           CLUSTER IS clus;
ANALYSIS:  TYPE IS TWOLEVEL;
MODEL:     %WITHIN%
           fw1 BY y1-y3;
           fw2 BY y4-y6;
           fw1 ON x1 x2;
           fw2 ON x1 x2;
           %BETWEEN%
           fb BY y1-y6;
           fb ON w;
```


Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates

```
TITLE:        This is an example of a two-level CFA with
              continuous factor indicators with two
              factors on the within level and one factor
              on the between level
MONTECARLO:
              NAMES ARE y1-y6 x1 x2 w;
              NOBSERVATIONS = 1000;
              NCSIZES = 3;
              CSIZES = 40 (5) 50 (10) 20 (15);
              SEED = 58459;
              NREPS = 1;
              SAVE = ex9.8.dat;
              WITHIN = x1 x2;
              BETWEEN = w;
ANALYSIS:     TYPE = TWOLEVEL;
```

Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates (Continued)

```
MODEL POPULATION:
              %WITHIN%
              x1-x2@1;
              fw1 BY y1@1 y2-y3*1;
              fw2 BY y4@1 y5-y6*1;
              fw1-fw2*1;
              y1-y6*1;
              fw1 ON x1*.5 x2*.7;
              fw2 ON x1*.7 x2*.5;

              %BETWEEN%
              [w@0]; w*1;
              fb BY y1@1 y2-y6*1;
              y1-y6*.3;
              fb*.5;
              fb ON w*1;
```

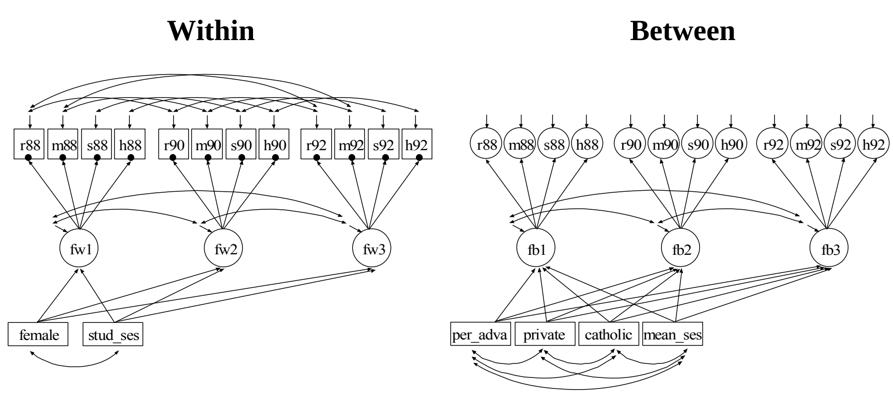

Input For Monte Carlo Simulations For Two-Level Factor Analysis With Covariates (Continued)

```
MODEL:
              %WITHIN%

              fw1 BY y1@1 y2-y3*1;
              fw2 BY y4@1 y5-y6*1;
              fw1-fw2*1;
              y1-y6*1;
              fw1 ON x1*.5 x2*.7;
              fw2 ON x1*.7 x2*.5;

              %BETWEEN%

              fb BY y1@1 y2-y6*1;
              y1-y6*.3;
              fb*.5;
              fb ON w*1;
OUTPUT:
              TECH8 TECH9;
```

NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school, n = 14,217
  - Variables—reading, math, science, history-citizenship-geography, and background variables
- Data for the analysis—reading, math, science, history-citizenship-geography

NELS Two-Level Longitudinal Factor Analysis With Covariates



Input For NELS Two-Level Longitudinal Factor Analysis With Covariates

```
TITLE:      two-level factor analysis with covariates using the NELS
            data
DATA:       FILE = NELS.dat;
            FORMAT = 2f7.0 f11.4 12f5.2 11f8.2;
VARIABLE:   NAMES = id school f2pnlwt r88 m88 s88 h88 r90 m90 s90 h90
            r92 m92 s92 h92 stud_ses female per_mino urban size rural
            private mean_ses catholic stu_teac per_adva;
                !Variable Description
                !m88 = math IRT score in 1988
                !m90 = math IRT score in 1990
                !m92 = math IRT score in 1992
                !r88 = reading IRT score in 1988
                !r90 = reading IRT score in 1990
                !r92 = reading IRT score in 1992
```


Input For NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
                !s88 = science IRT score in 1988
                !s90 = science IRT score in 1990
                !s92 = science IRT score in 1992
                !h88 = history IRT score in 1988
                !h90 = history IRT score in 1990
                !h92 = history IRT score in 1992
                !female = scored 1 vs 0
                !stud_ses = student family ses in 1990 (f1ses)
                !per_adva = percent teachers with an MA or higher
                !private = private school (scored 1 vs 0)
                !catholic = catholic school (scored 1 vs 0)
                !private = 0, catholic = 0 implies public school
            MISSING = BLANK;
            CLUSTER = school;
            USEV = r88 m88 s88 h88 r90 m90 s90 h90 r92 m92 s92 h92
            female stud_ses per_adva private catholic mean_ses;
            WITHIN = female stud_ses;
            BETWEEN = per_adva private catholic mean_ses;
```

Input For NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
ANALYSIS:    TYPE = TWOLEVEL;
MODEL:       %WITHIN%
             fw1 BY r88-h88;
             fw2 BY r90-h90;
             fw3 BY r92-h92;
             r88 WITH r90; r90 WITH r92; r88 WITH r92;
             m88 WITH m90; m90 WITH m92; m88 WITH m92;
             s88 WITH s90; s90 WITH s92;
             h88 WITH h90; h90 WITH h92;
             fw1-fw3 ON female stud_ses;

             %BETWEEN%
             fb1 BY r88-h88;
             fb2 BY r90-h90;
             fb3 BY r92-h92;
             fb1-fb3 ON per_adva private catholic mean_ses;
OUTPUT:      SAMPSTAT STANDARDIZED TECH1 TECH8 MODINDICES;
```


Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates

Summary Of Data

```
    Number of patterns      15
    Number of clusters     913

Average cluster size 15.572

    Estimated Intraclass Correlations for the Y Variables
```

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
R88	0.067	M88	0.129	S88	0.100
H88	0.105	R90	0.076	M90	0.117
S90	0.110	H90	0.106	R92	0.073
M92	0.111	S92	0.099	H92	0.091

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Tests Of Model Fit

```
Chi-Square Test of Model Fit
              Value                                  4883.539*
              Degrees of Freedom                          146
              P-Value                                  0.0000
              Scaling Correction Factor                 1.046
                 for MLR

Chi-Square Test of Model Fit for the Baseline Model
              Value                                150256.855
              Degrees of Freedom                          202
              P-Value                                  0.0000

CFI/TLI
              CFI                                       0.968
              TLI                                       0.956

Loglikelihood
              H0 Value                            -487323.777
              H1 Value                            -484770.257
```


Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

```
Information Criteria

              Number of Free Parameters                    94
              Akaike (AIC)                         974835.554
              Bayesian (BIC)                       975546.400
              Sample-Size Adjusted BIC             975247.676
                (n* = (n + 2) / 24)

RMSEA (Root Mean Square Error Of Approximation)
              Estimate                                  0.048
SRMR (Standardized Root Mean Square Residual
              Value for Between                         0.041
              Value for Within                          0.027
```

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Model Results

		Estimates	S.E.	Est./S.E.	Std	StdYX
Within Level						
FW1	BY					
	R88	1.000	0.000	0.000	6.528	0.812
	M88	0.940	0.010	94.856	6.135	0.804
	S88	1.005	0.010	95.778	6.559	0.837
	H88	1.041	0.011	97.888	6.796	0.837
FW2	BY					
	R90	1.000	0.000	0.000	8.038	0.842
	M90	0.911	0.008	109.676	7.321	0.838
	S90	1.003	0.010	99.042	8.065	0.859
	H90	0.939	0.008	113.603	7.544	0.855

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

FW3	BY					
	R92	1.000	0.000	0.000	8.460	0.832
	M92	0.939	0.009	101.473	7.946	0.845
	S92	1.003	0.011	90.276	8.482	0.861
	H92	0.934	0.009	102.825	7.905	0.858
FW1	ON					
	FEMALE	-0.403	0.128	-3.150	-0.062	-0.031
	STUD_SES	3.378	0.096	35.264	0.517	0.418
FW2	ON					
	FEMALE	-0.621	0.157	-3.945	-0.077	-0.039
	STUD_SES	4.169	0.110	37.746	0.519	0.420
FW3	ON					
	FEMALE	-1.027	0.169	-6.087	-0.121	-0.064
	STUD_SES	4.418	0.122	36.124	0.522	0.422

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Residual Variances

R88	22.021	0.383	57.464	22.021	0.341
M88	20.618	0.338	61.009	20.618	0.354
S88	18.383	0.323	56.939	18.383	0.299
H88	19.805	0.370	53.587	19.805	0.300
R90	26.546	0.491	54.033	26.546	0.291
M90	22.756	0.375	60.748	22.756	0.298
S90	23.150	0.383	60.516	23.150	0.262
H90	21.002	0.403	52.124	21.002	0.270
R92	31.821	0.617	51.562	31.821	0.308
M92	25.213	0.485	52.018	25.213	0.285
S92	25.155	0.524	47.974	25.155	0.259
H92	22.479	0.489	46.016	22.479	0.265
FW1	35.081	0.699	50.201	0.823	0.823
FW2	53.079	1.005	52.806	0.822	0.822
FW3	58.438	1.242	47.041	0.817	0.817

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Between Level

FB1 BY					
R88	1.000	0.000	0.000	1.952	0.933
M88	1.553	0.070	22.138	3.031	0.979
S88	1.061	0.058	18.255	2.071	0.887
H88	1.065	0.053	19.988	2.078	0.814
FB2 BY					
R90	1.000	0.000	0.000	2.413	0.923
M90	1.407	0.058	24.407	3.395	1.003
S90	1.220	0.062	19.697	2.943	0.946
H90	0.973	0.047	20.496	2.348	0.829
FB3 BY					
R92	1.000	0.000	0.000	2.472	0.947
M92	1.435	0.065	22.095	3.546	0.997
S92	1.160	0.065	17.889	2.868	0.938
H92	0.963	0.041	23.244	2.380	0.871

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

Between Level

FB1 ON					
PER_ADVA	0.217	0.292	0.742	0.111	0.024
PRIVATE	0.303	0.344	0.883	0.155	0.042
CATHOLIC	-0.696	0.277	-2.512	-0.357	-0.088
MEAN_SES	2.513	0.206	12.185	1.288	0.672
FB2 ON					
PER_ADVA	0.280	0.338	0.828	0.116	0.025
PRIVATE	0.453	0.392	1.155	0.188	0.051
CATHOLIC	-0.538	0.334	-1.609	-0.223	-0.055
MEAN_SES	3.054	0.239	12.805	1.266	0.660
FB3 ON					
PER_ADVA	0.473	0.375	1.261	0.192	0.041
PRIVATE	0.673	0.435	1.547	0.272	0.074
CATHOLIC	-0.206	0.372	-0.554	-0.084	-0.021
MEAN_SES	3.142	0.258	12.169	1.271	0.663

Output Excerpts NELS Two-Level Longitudinal Factor Analysis With Covariates (Continued)

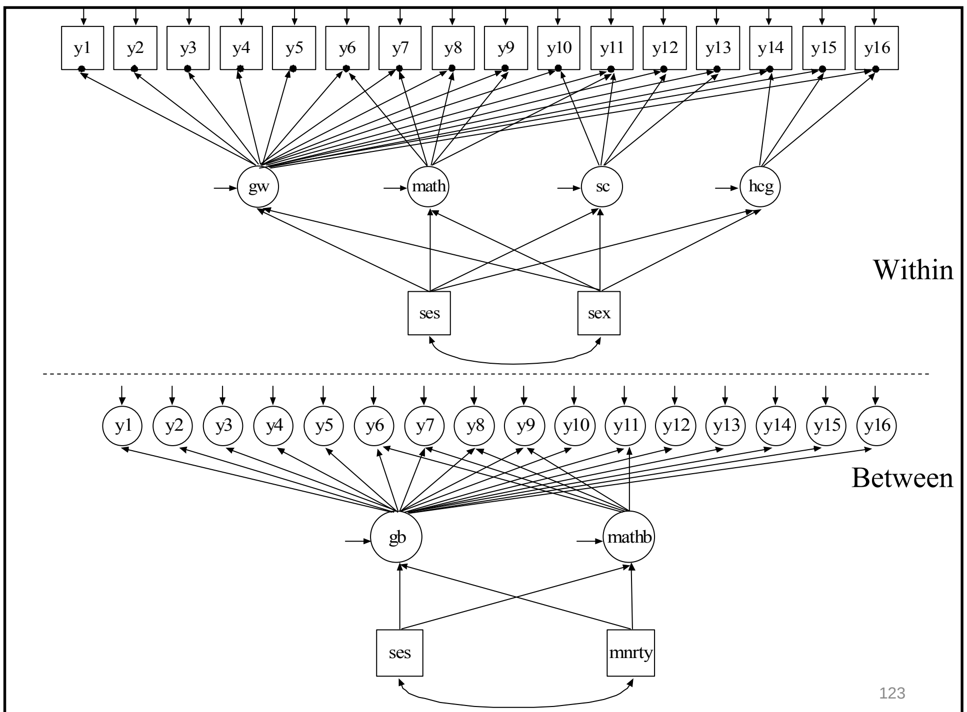
Residual Variances

R88	0.564	0.104	5.437	0.564	0.129
M88	0.399	0.093	4.292	0.399	0.042
S88	1.160	0.126	9.170	1.160	0.213
H88	2.203	0.203	10.839	2.203	0.338
R90	1.017	0.160	6.352	1.017	0.149
M90	-0.068	0.055	-1.225	-0.068	-0.006
S90	1.025	0.172	5.945	1.025	0.106
H90	2.518	0.216	11.636	2.518	0.313
R92	0.706	0.182	3.886	0.706	0.104
M92	0.076	0.076	1.000	0.076	0.006
S92	1.120	0.190	5.901	1.120	0.120
H92	1.810	0.211	8.599	1.810	0.242
FB1	1.979	0.245	8.066	0.520	0.520
FB2	3.061	0.345	8.875	0.526	0.526
FB3	3.010	0.409	7.363	0.493	0.493

Multiple-Group, Two-Level Factor Analysis With Covariates

NELS Data

- The data—National Education Longitudinal Study (NELS:88)
  - Base year Grade 8—followed up in Grades 10 and 12
  - Students sampled within 1,035 schools—approximately 26 students per school
  - Variables—reading, math, science, history-citizenship-geography, and background variables
- Data for the analysis—reading, math, science, history-citizenship-geography, gender, individual SES, school SES, and minority status, n = 14,217 with 913 schools (clusters)



Input For NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools

```
TITLE:     NELS:88 with listwise deletion
           disaggregated model for two groups, public and
           catholic schools
DATA:      FILE IS EX831.DAT;;
VARIABLE:  NAMES = ses y1-y16 gender cluster minority group;
           CLUSTER = cluster;
           WITHIN = gender;
           BETWEEN = minority;
           GROUPING = group(1=public 2=catholic);
DEFINE:    minority = minority/5;
ANALYSIS:  TYPE = TWOLEVEL;
           H1ITER = 2500;
           MITER = 1000;
```

Input For NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

```
MODEL:     %WITHIN%
                 generalw BY y1* y2-y6 y8-y16 y7@1;

                 mathw BY y6* y8* y9* y11 y7@1;
                 scw BY y10 y11*.5 y12*.3 y13*.2;
                 hcgw BY y14*.7 y16*2 y15@1;

                 generalw WITH mathw-hcgw@0;
                 mathw WITH scw-hcgw@0;
                 scw WITH hcgw@0;

                 generalw mathw scw hcgw ON gender ses;
           %BETWEEN%
                 generalb BY y1* y2-y6 y8-y16 y7@1;
                 mathb BY y6* y8 y9 y11 y7@1;

                 y1-y16@0;

                 generalb WITH mathb@0;

                 generalb mathb ON ses minority;
```


Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools

Summary Of Data

```
Group PUBLIC
   Number of clusters        195
     Size (s) Cluster ID with Size s
       1        68114   68519
       2        72872
       7        72765
       8        45991   72012
       9        68071
      10         7298   72187
      11        72463    7105   72405
      12        24083   68971    7737   68390
      13        45861   72219   72049
      14        68511   72148   72175   72176  25464
      15        68023   25071   68748   45928   7915   78324
      16        45362    7403   72415   77204  77219   72456
      17        45502   68487   45824    7203  24948    7829   72612    7892
                25835    7591   68155   68295
```

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

18	72133	25580	24910	68614	25074	72990	68328	25404
	7348							
19	7671	68662	68671	45385	7438	7332	25615	72799
	68340	72956	25642	25658	24856	78283	68030	
20	72617	72715	7211	25422	7330	72292	72060	72993
	7451	68461	78162	78232	72170	25130		
21	45394	7193	68180	24589	7205	25894	25958	68391
	77254	77634	68448	45271	7584	25227	78598	
22	68254	68397	68648	72768	7192	7117	7119	68753
	24813							
23	68456	25361	7157	25702	25804	45620	24858	7658
	25163	45041	77351	45183	77684	78101	68788	68817
	7792	78311	68048	68453				
24	77222	24053	7000	77403	24138	68297	78011	25536
	7778	72042	25360	25977	45747	7616	78886	
25	68906	68720	25354	68427	72833	77268	7269	68520
	77537	72075						
26	72973	45555	24828	68315	45087	25328	77710	25848
27	45831	25618	68652	72080	45900	25208	45452	7103

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

28	25666	68809	25076	25224	68551
30	7343	45978	25722	45924	
31	77109	7230	68855		
32	25178				
33	45330	25745	25825		
35	25667				
36	72129				
37	25834				
38	45287				
39	45197	7090			
43	45366				

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group PUBLIC

Number of clusters 195
Average cluster size 21.292

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
Y1	.111	Y7	.100	Y12	.115
Y2	.105	Y8	.124	Y13	.185
Y3	.213	Y9	.069	Y14	.094
Y4	.160	Y10	.147	Y15	.132
Y5	.081	Y11	.105	Y16	.159
Y6	.159				

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Group CATHOLIC

Number of clusters 40
Average cluster size 26.016

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
Y1	.010	Y7	.029	Y12	.056
Y2	.039	Y8	.061	Y13	.176
Y3	.180	Y9	.056	Y14	.078
Y4	.091	Y10	.079	Y15	.071
Y5	.055	Y11	.056	Y16	.154
Y6	.118				

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

Tests Of Model Fit

```
Loglikelihood
              Value                          1716.922*
              Degrees of Freedom                   575
              P-Value                           0.0000
              Scaling Correction Factor          0.872
                 for MLR

Chi-Square Test of Model
              Value                          35476.471
              Degrees of Freedom                   608
              P-Value                           0.0000

CFI/TLI
              CFI                                0.967
              TLI                                0.965

Loglikelihood
              H0 Value                     -130332.921
              H1 Value                     -129584.053
```


Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group Public					
Within Level					
GENERALW ON					
GENDER	-0.193	0.029	-6.559	-0.256	-0.128
SES	0.233	0.016	14.269	0.309	0.279
MATHW ON					
GENDER	0.266	0.025	10.534	0.510	0.255
SES	0.054	0.014	3.879	0.103	0.093
SCW ON					
GENDER	0.452	0.032	14.005	0.961	0.480
SES	0.018	0.015	1.244	0.039	0.035
HCGW ON					
GENDER	0.152	0.023	6.588	0.681	0.341
SES	0.002	0.007	0.239	0.007	0.007

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group Catholic					
Within Level					
GENERALW ON					
GENDER	-0.294	0.059	-5.000	-0.403	-0.201
SES	0.169	0.021	7.892	0.232	0.193
MATHW ON					
GENDER	0.332	0.051	6.478	0.627	0.313
SES	-0.030	0.017	-1.707	-0.056	-0.047
SCW ON					
GENDER	0.555	0.063	8.860	1.226	0.613
SES	-0.022	0.014	-1.592	-0.049	-0.041
HCGW ON					
GENDER	0.160	0.029	5.610	0.785	0.392
SES	0.001	0.007	0.089	0.003	0.002

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group Public					
Between Level					
GENERALB ON					
SES	0.505	0.079	6.390	1.244	0.726
MINORITY	-0.217	0.088	-2.452	-0.534	-0.188
MATHB ON					
SES	0.198	0.070	2.825	0.984	0.574
MINORITY	-0.031	0.087	-0.354	-0.153	-0.054
GENERALB WITH					
MATHB	0.000	0.000	0.000	0.000	0.000
Intercepts					
GENERALB	0.000	0.000	0.000	0.000	0.000
MATHB	0.000	0.000	0.000	0.000	0.000

Output Excerpts NELS:88 Two-Group, Two-Level Model For Public And Catholic Schools (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
Group Catholic					
Between Level					
GENERALB ON					
SES	0.262	0.067	3.929	0.975	0.538
MINORITY	-0.327	0.069	-4.707	-0.216	-0.573
MATHB ON					
SES	0.205	0.071	2.901	0.746	0.412
MINORITY	-0.213	0.095	-2.241	-0.778	-0.367
GENERALB WITH					
MATHB	0.000	0.000	0.000	0.000	0.000
Intercepts					
GENERALB	0.466	0.163	2.854	1.734	1.734
MATHB	0.573	0.177	3.239	2.087	2.087

Further Readings On Two-Level Factor Analysis

Harnqvist, K., Gustafsson, J.E., Muthén, B, & Nelson, G. (1994). Hierarchical models of ability at class and individual levels. Intelligence, 18, 165-187. (#53)

Hox, J. (2002). Multilevel analysis. Techniques and applications. Mahwah, NJ: Lawrence Erlbaum

Longford, N. T., & Muthén, B. (1992). Factor analysis for clustered observations. Psychometrika, 57, 581-597. (#41)

Muthén, B. (1989). Latent variable modeling in heterogeneous populations. Psychometrika, 54, 557-585. (#24)

Muthén, B. (1990). Mean and covariance structure analysis of hierarchical data. Paper presented at the Psychometric Society meeting in Princeton, NJ, June 1990. UCLA Statistics Series 62. (#32)

Muthén, B. (1991). Multilevel factor analysis of class and student achievement components. Journal of Educational Measurement, 28, 338-354. (#37)

Further Readings On Two-Level Factor Analysis (Continued)

Muthén, B. (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research, 22, 376-398. (#55)

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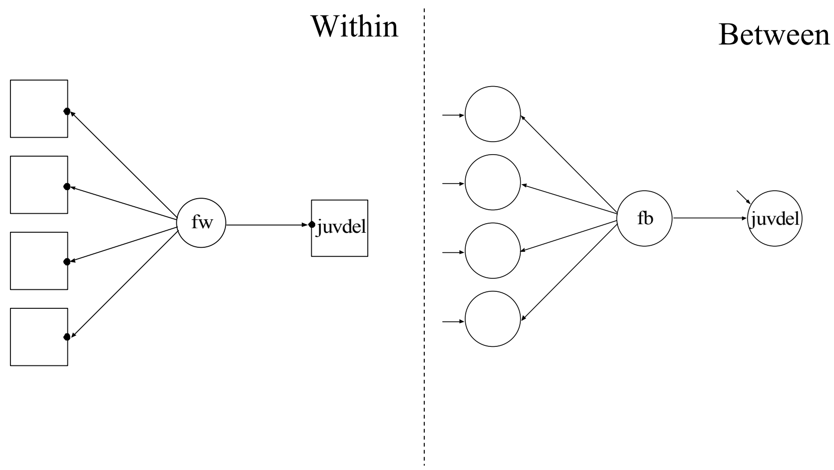
Two-Level Structural Equation Modeling

Predicting Juvenile Delinquency From First Grade Aggressive Behavior. Two-Level Logistic Regression On A Factor



Input Excerpts Two-Level Logistic Regression On A Factor

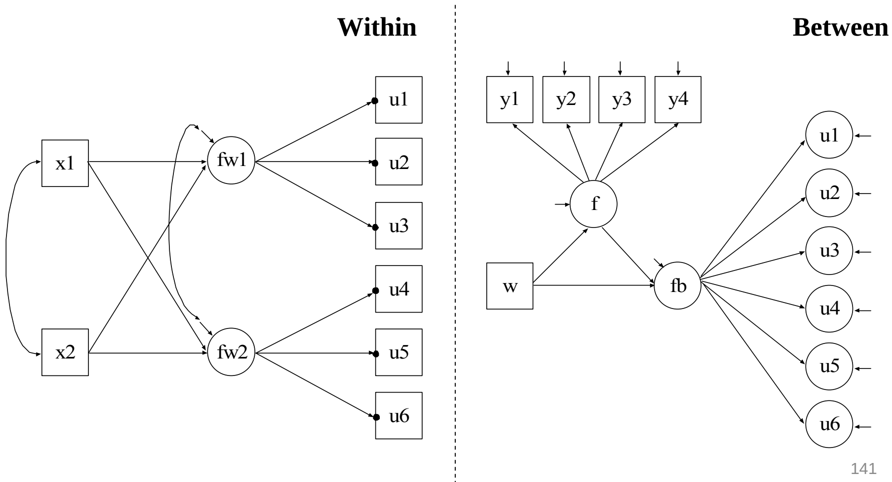
```
VARIABLE:     CLUSTER=classrm;
              USEVAR = juv99 gender stub1F bkRule1F harm01F
              bkThin1F yell1F takeP1F fight1F lies1F tease1F;
              CATEGORICAL = juv99;
              MISSING = ALL (999);
              WITHIN = gender;

ANALYSIS:     TYPE = TWOLEVEL;

MODEL:        %WITHIN%
              fw BY stub1F bkRule1F harm01F bkThin1F yell1F
              takeP1F fight1F lies1F tease1F;
              juv99 ON gender fw;
              %BETWEEN%
              fb BY stub1F bkRule1F harm01F bkThin1F yell1F
              takeP1F fight1F lies1F tease1F;
              juv99 ON fb;

OUTPUT:       TECH1 TECH8;
```

Two-Level SEM With Categorical Factor Indicators On The Within Level And Cluster-Level Continuous Observed And Random Intercept Factor Indicators On the Between Level



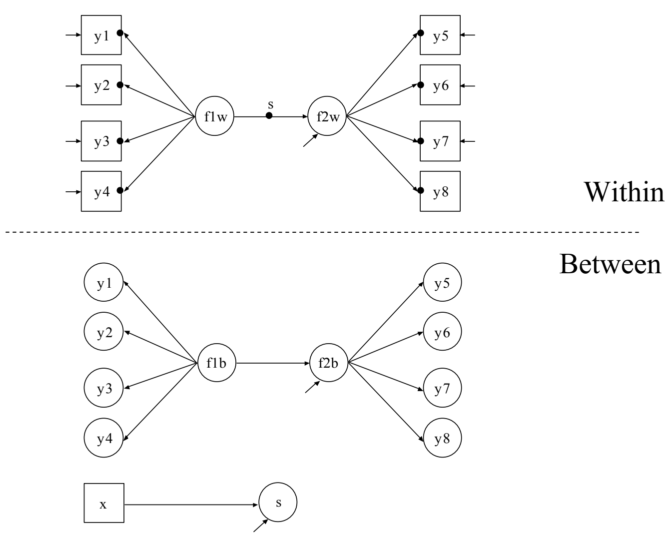
Two-Level SEM With Categorical Factor Indicators On The Within Level And Cluster-Level Continuous Observed And Random Intercept Factor Indicators On the Between Level

```
TITLE:          this is an example of a two-level SEM with
                categorical factor indicators on the within level
                and cluster-level continuous observed and random
                intercept factor indicators on the between level
DATA:           FILE IS ex9.9.dat;
VARIABLE:       NAMES ARE u1-u6 y1-y4 x1 x2 w clus;
                CATEGORICAL = u1-u6;
                WITHIN = x1 x2;
                BETWEEN = w y1-y4;
                CLUSTER IS clus;
ANALYSIS:       TYPE IS TWOLEVEL;
                ESTIMATOR = WLSMV;
MODEL:
                %WITHIN%
                fw1 BY u1-u3;
                fw2 BY u4-u6;
                fw1 fw2 ON x1 x2;
```

Two-Level SEM With Categorical Factor Indicators On The Within Level And Cluster-Level Continuous Observed And Random Intercept Factor Indicators On the Between Level

```
                %BETWEEN%
                fb BY u1-u6;
                f BY y1-y4;
                fb ON w f;
                f ON w;
SAVEDATA:       SWMATRIX = ex9.9sw.dat;
```


Two-Level SEM: Random Slopes For Regressions Among Factors



Two-Level Estimators In Mplus

- Maximum-likelihood:
  - Outcomes: Continuous, censored, binary, ordered and unordered categorical, counts and combinations
  - Random intercepts and slopes; individually-varying times of observation; random slopes for time-varying covariates; random slopes for dependent variables; random slopes for latent independent and dependent variables
  - Missing data
- Limited information weighted least-squares:
  - Outcomes: Continuous, categorical, and combinations
  - Random intercepts
  - Missing data
- Muthen's limited information estimator (MUML):
  - Outcomes: Continuous
  - Random intercepts
  - No missing data

Non-normality robust SEs and chi-square test of model fit.

Practical Issues Related To The Analysis Of Multilevel Data

Size Of The Intraclass Correlation

- The importance of the size of an intraclass correlation depends on the size of the clusters
- Small intraclass correlations can be ignored but important information about between-level variability may be missed by conventional analysis
- Intraclass correlations are attenuated by individual-level measurement error
- Effects of clustering not always seen in intraclass correlations

Practical Issues Related To The Analysis Of Multilevel Data (Continued)

Sample Size

- There should be at least 30-50 between-level units (clusters)
- Clusters with only one observation are allowed
- More clusters than between-level parameters

Steps In SEM Multilevel Analysis For Continuous Outcomes

1) Explore SEM model using the sample covariance matrix from the total sample
2) Estimate the SEM model using the pooled-within sample covariance matrix with sample size n - G
3) Investigate the size of the intraclass correlations and DEFF's
4) Explore the between structure using the estimated between covariance matrix with sample size G
5) Estimate and modify the two-level model suggested by the previous steps

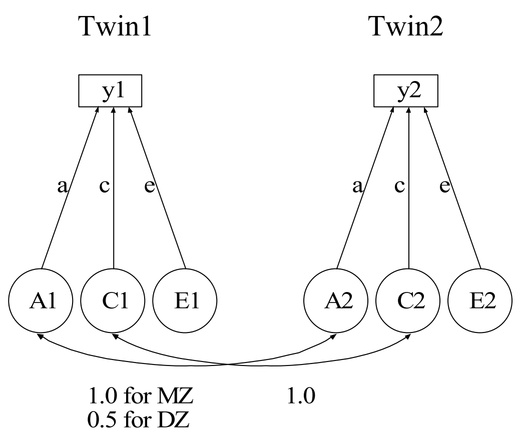
Muthén, B. (1994). Multilevel covariance structure analysis. In J. Hox & I. Kreft (eds.), Multilevel Modeling, a special issue of Sociological Methods & Research, 22, 376-398. (#55)

Multivariate Approach To Multilevel Modeling

Multivariate Modeling Of Family Members

- Multilevel modeling: clusters independent, model for between- and within-cluster variation, units within a cluster statistically equivalent
- Multivariate approach: clusters independent, model for all variables for each cluster unit, different parameters for different cluster units.
  - Used in latent variable growth modeling where the cluster units are the repeated measures over time
  - Allows for different cluster sizes by missing data techniques
  - More flexible than the multilevel approach, but computationally convenient only for applications with small cluster sizes (e.g. twins, spouses)

Twin Modeling



Neale & Cardon (1992)
Prescott (2004)

Two-Level Mixture Modeling: Within-Level Latent Classes

Regression Mixture Analysis

Two-Level Regression Mixture Model

$$y_{ij\,|\,Cij=c} = \beta_{0cj} + \beta_{1cj}\,x_{ij} + r_{ij}\,, \qquad (3)$$

$$P(C_{ij} = c \mid z_{ij}) = \frac{e^{a_{cj} + b_c z_{ij}}}{\sum_{s=1}^{K} e^{a_{sj} + b_s z_{ij}}} \qquad (4)$$

$$\beta_{0cj} = \gamma_{00c} + \gamma_{01c} w_{0j} + u_{0j}\,, \qquad (5)$$

$$\beta_{1cj} = \gamma_{10c} + \gamma_{11c} w_{1j} + u_{1j}\,, \qquad (6)$$

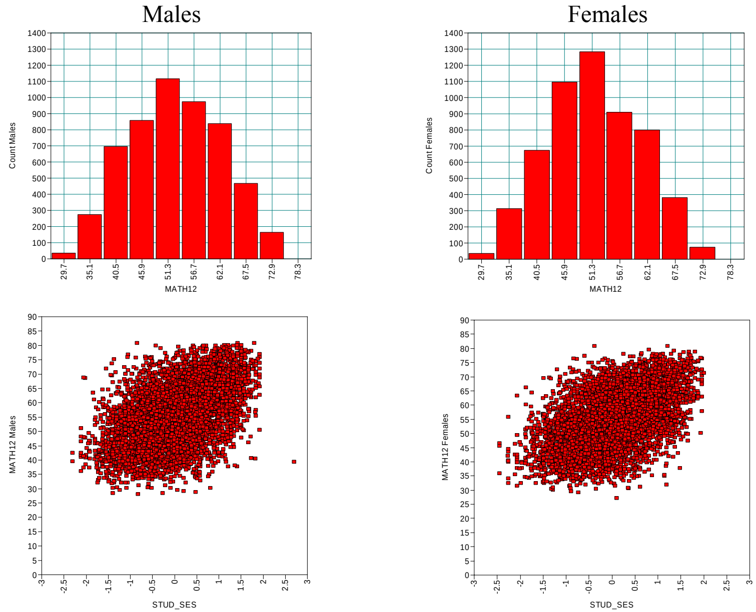
$$a_{cj} = \gamma_{20c} + \gamma_{21c} w_{2j} + u_{2cj} \qquad (7)$$

Muthén & Asparouhov (2009), JRSS-A

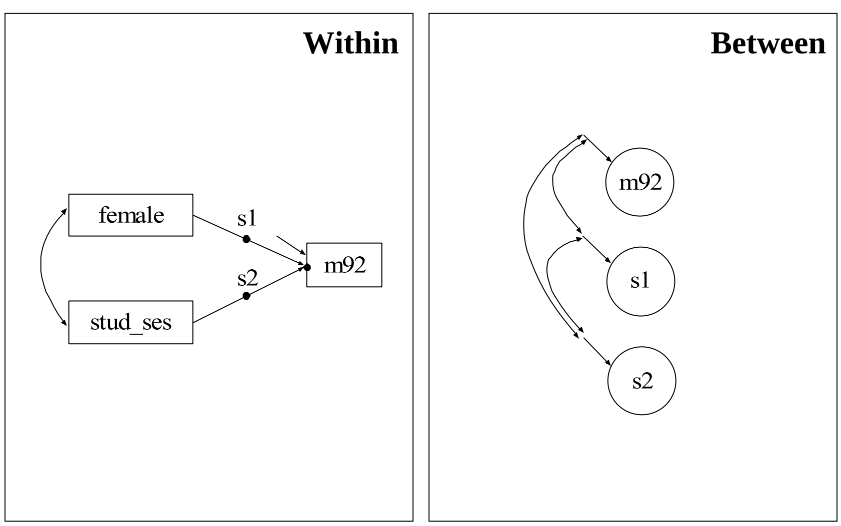
Two-Level Data

- Education studies of students within schools
  - LSAY (3,000 students in 54 schools, grades 7-12)
  - NELS (14,000 students in 900 schools, grades 8-12),
  - ECLS (22,000 students in 1,000 schools, K- grade 8)
- Public health studies of patients within hospitals, individuals within counties

NELS Data: Grade 12 Math Related To Gender And SES



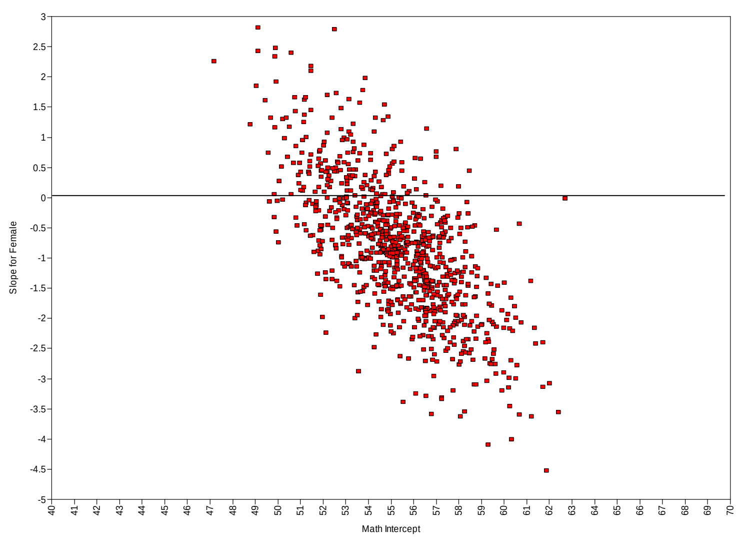
NELS Two-Level Math Achievement Regression



Output Excerpts NELS Two-Level Regression

	Estimates	S.E.	Est./S.E.
Between Level			
Means			
M92	55.279	0.174	317.706
S_FEMALE	-0.850	0.188	-4.507
S_SES	5.450	0.132	41.228
Variances			
M92	11.814	1.197	9.870
S_FEMALE	5.762	1.426	4.041
S_SES	0.905	0.538	1.682
S_FEMALE WITH			
M92	-4.936	1.071	-4.610
S_SES	0.068	0.635	0.107
S_SES WITH			
M92	1.314	0.541	2.431

Random Effect Estimates For Each School: Slopes For Female Versus Intercepts For Math

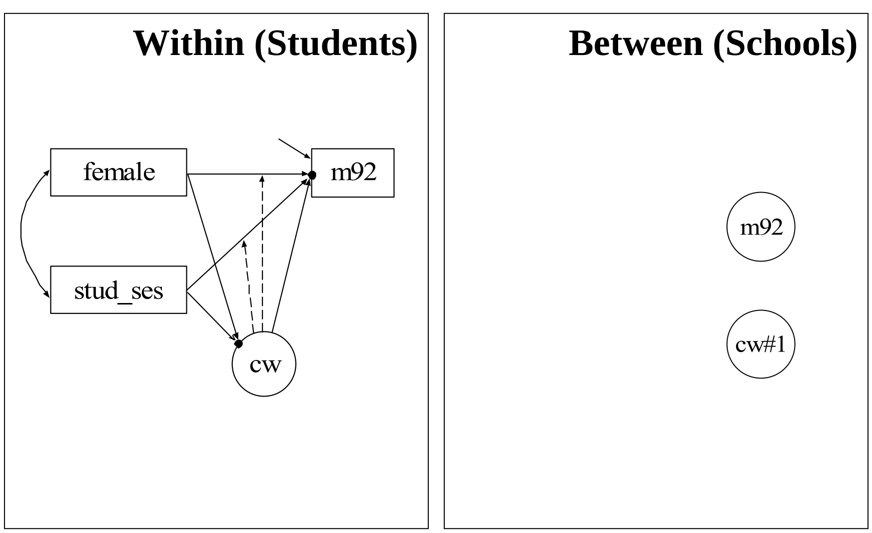


Is The Conventional Two-Level Regression Model Sufficient?

- Conventional Two-Level Regression of Math Score Related to Gender and Student SES
  - Loglikelihood = -39,512, number of parameters = 10, BIC = 79,117
- New Model
  - Loglikelihood = -39,368, number of parameters = 12, BIC = 78,848

  - Which model would you choose?

Two-Level Regression With Latent Classes For Students

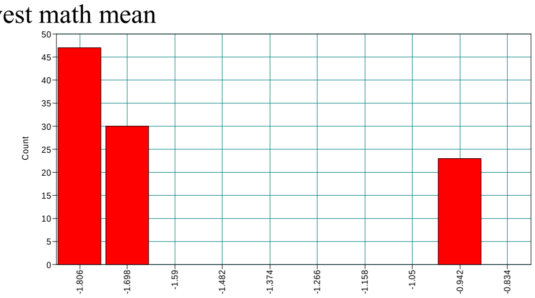


Model Results For NELS Two-Level Regression Of Math Score Related To Gender And Student SES

Model	Loglikelihood	# parameters	BIC
(1) Conventional 2-level regression with random intercepts and random slopes	-39,512	10	79,117
(2) Two-level regression mixture, 2 latent classes for students	-39,368	12	78,848
(3) Two-level regression mixture, 3 latent classes for students	**-39,280**	**19**	**78,736**

Estimated Two-Level Regression Mixture With 3 Latent Classes For Students

- Estimated Female slope means for the 3 latent classes for students do not include positive values.
- The class with the least Female disadvantage (right-most bar) has the lowest math mean



- Significant between-level variation in cw (the random mean of the latent class variable for students): Schools have a significant effect on latent class membership for students

Input For Two-Level Regression With Latent Classes For Students

```
TITLE:    NELS 2-level regression
DATA:     FILE = comp.dat;
          FORMAT = 2f7.0 f11.4 13f5.2 79f8.2 f11.7;
VARIABLE:
          NAMES = school m92 female stud_ses;
          CLUSTER = school;
          USEV = m92 female stud_ses;
          WITHIN = female stud_ses;
          CENTERING = GRANDMEAN(stud_ses);
          CLASSES = cw(3);
ANALYSIS:
          TYPE = TWOLEVEL MIXTURE;
          PROCESS = 2;
          INTERACTIVE = control.dat;
          !STARTS = 1000 100;
          STARTS = 0;
```


Input For Two-Level Regression With Latent Classes For Students (Continued)

```
MODEL:
           %WITHIN%
           %OVERALL%
           m92 ON female stud_ses;
           cw#1-cw#2 ON female stud_ses;
   ! [m92] class-varying by default
           %cw#1%
           m92 ON female stud_ses;
           %cw#2%
           m92 ON female stud_ses;
           %cw#3%
           m92 ON female stud_ses;
           %BETWEEN%
           %OVERALL%
           f BY cw#1 cw#2;
```

Cluster-Randomized Trials And NonCompliance

Randomized Trials With NonCompliance

- Tx group (compliance status observed)
  - Compliers
  - Noncompliers
- Control group (compliance status unobserved)
  - Compliers
  - NonCompliers

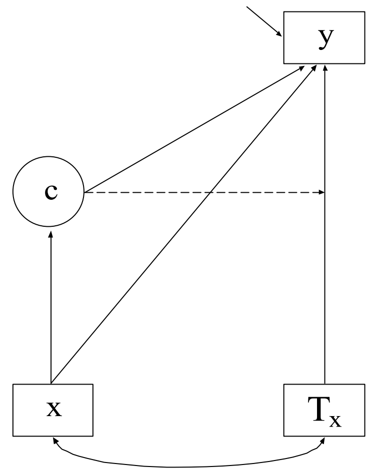
Compliers and Noncompliers are typically not randomly equivalent subgroups.

Four approaches to estimating treatment effects:

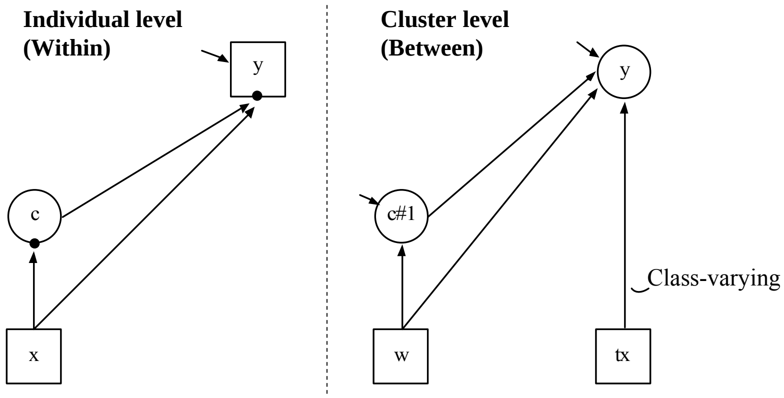
1. Tx versus Control (Intent-To-Treat; ITT)
2. Tx Compliers versus Control (Per Protocol)
3. Tx Compliers versus Tx NonCompliers + Control (As-Treated)
4. Mixture analysis (Complier Average Causal Effect; CACE):
   - Tx Compliers versus Control Compliers
   - Tx NonCompliers versus Control NonCompliers

CACE: Little & Yau (1998) in Psychological Methods

Randomized Trials with NonCompliance: Complier Average Causal Effect (CACE) Estimation



Two-Level Regression Mixture Modeling: Cluster-Randomized CACE



Further Readings On Non-Compliance Modeling

Dunn, G., Maracy, M., Dowrick, C., Ayuso-Mateos, J.L., Dalgard, O.S., Page, H., Lehtinen, V., Casey, P., Wilkinson, C., Vasquez-Barquero, J.L., & Wilkinson, G. (2003). Estimating psychological treatment effects from a randomized controlled trial with both non-compliance and loss to follow-up. British Journal of Psychiatry, 183, 323-331.

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Further Readings On Non-Compliance Modeling: Two-Level Modeling

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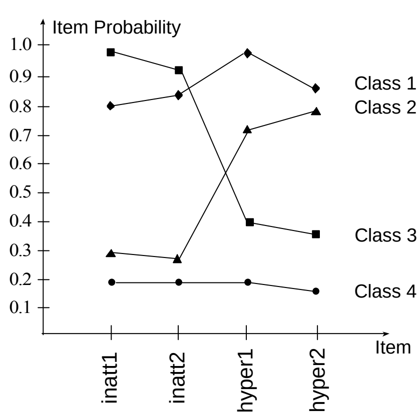
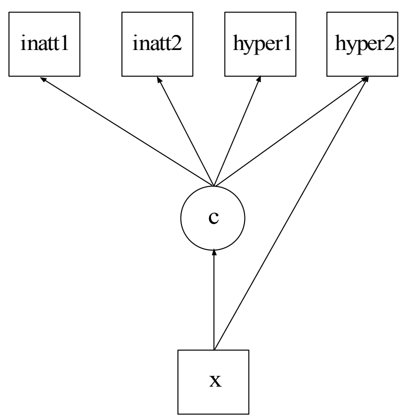
Jo, B., Asparouhov, T., Muthén, B. O., Ialongo, N. S., & Brown, C. H. (2008). Cluster Randomized Trials with Treatment Noncompliance. Psychological Methods, 13, 1-18.

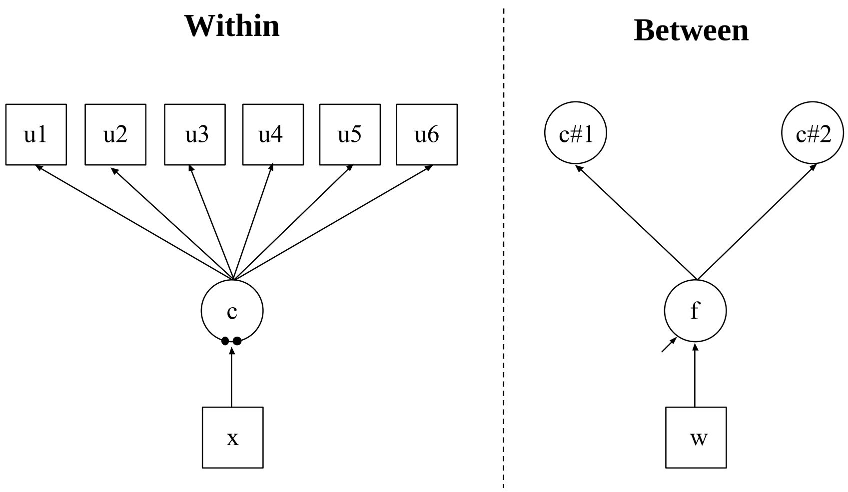
Latent Class Analysis

Latent Class Analysis





Two-Level Latent Class Analysis



Input For Two-Level Latent Class Analysis

```
TITLE:     this is an example of a two-level LCA with
           categorical latent class indicators
DATA:      FILE IS ex10.3.dat;
VARIABLE:  NAMES ARE u1-u6 x w c clus;
           USEVARIABLES = u1-u6 x w;
           CATEGORICAL = u1-u6;
           CLASSES = c (3);
           WITHIN = x;
           BETWEEN = w;
           CLUSTER = clus;
ANALYSIS:  TYPE = TWOLEVEL MIXTURE;
```

Input For Two-Level Latent Class Analysis (Continued)

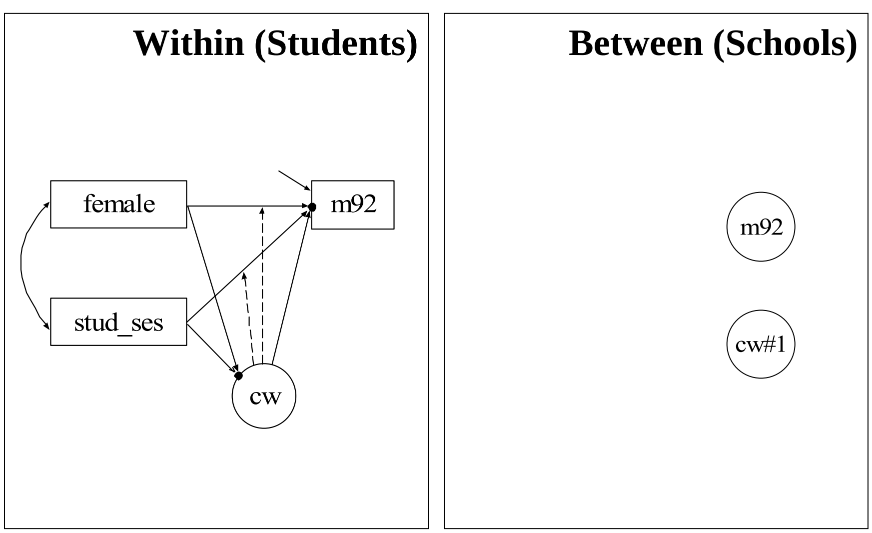
```
MODEL:      %WITHIN%
            %OVERALL%
            c#1 c#2 ON x;

            %BETWEEN%
            %OVERALL%
            f BY c#1 c#2;
            f ON w;
OUTPUT:     TECH1 TECH8;
```

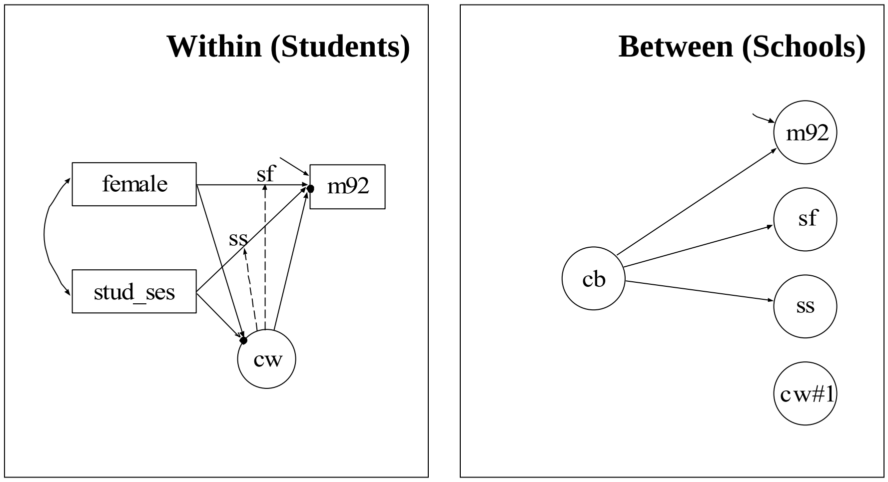

Two-Level Mixture Modeling: Between-Level Latent Classes

Regression Mixture Analysis

NELS Two-Level Regression With Latent Classes For Students



NELS Two-Level Regression With Latent Classes For Students And Schools

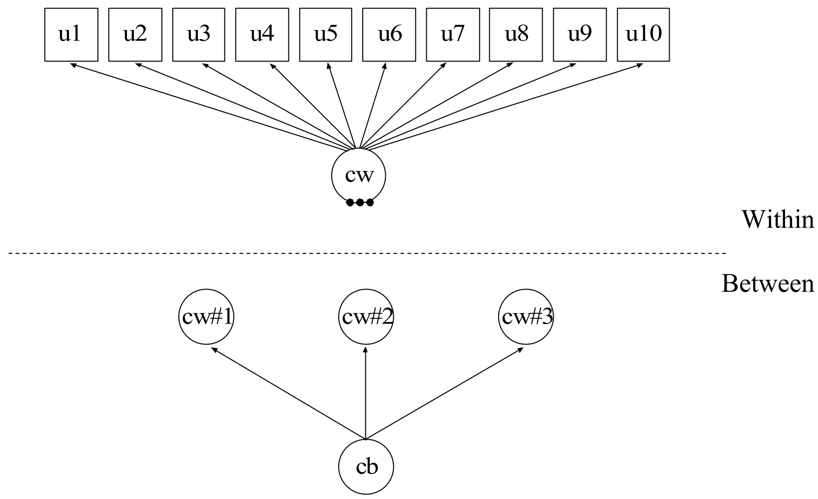


Model Results For NELS Two-Level Regression Of Math Score Related To Gender And Student SES

Model	Loglikelihood	# parameters	BIC
(1) Conventional 2-level regression with random intercepts and random slopes	-39,512	10	79,117
(2) Two-level regression mixture, 2 latent classes for students	-39,368	12	78,848
(3) Two-level regression mixture, 3 latent classes for students	**-39,280**	**19**	**78,736**
(4) Two-level regression mixture, 2 latent classes for schools, 2 latent classes for students	-39,348	19	78,873
(5) Two-level regression mixture, 2 latent classes for schools, 3 latent classes for students	-39,260	29	78,789

Latent Class Analysis

Two-Level LCA With Categorical Latent Class Indicators And A Between-Level Categorical Latent Variable



Input For Two-Level Latent Class Analysis

```
TITLE:          this is an example of a two-level LCA with
                categorical latent class indicators and a between-
                level categorical latent variable
DATA:           FILE = ex4.dat;
VARIABLE:       NAMES ARE u1-u10 dumb dumw clus;
                USEVARIABLES = u1-u10;
                CATEGORICAL = u1-u10;
                CLASSES = cb(5) cw(4);
                WITHIN = u1-u10;
                BETWEEN = cb;
                CLUSTER = clus;
ANALYSIS:       TYPE = TWOLEVEL MIXTURE;
                PROCESSORS = 2;
                STARTS = 100 10;
MODEL:
                %WITHIN%
                %OVERALL%
                %BETWEEN%
                %OVERALL%
                cw#1-cw#3 ON cb#1-cb#4;
```


Input For Two-Level Latent Class Analysis (Continued)

```
MODEL cw:
            %WITHIN%
            %cw#1%
            [u1$1-u10$1];
            [u1$2-u10$2];
            %cw#2%
            [u1$1-u10$1];
            [u1$2-u10$2];
            %cw#3%
            [u1$1-u10$1];
            [u1$2-u10$2];
            %cw#4%
            [u1$1-u10$1];
            [u1$2-u10$2];
OUTPUT:     TECH1 TECH8;
```

References

(To request a Muthén paper, please email bmuthen@ucla.edu.)

Cross-sectional Data

Asparouhov, T. (2005). Sampling weights in latent variable modeling. Structural Equation Modeling, 12, 411-434.

Asparouhov, T. & Muthén, B. (2007). Computationally efficient estimation of multilevel high-dimensional latent variable models. Proceedings of the 2007 JSM meeting in Salt Lake City, Utah, Section on Statistics in Epidemiology.

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